Variable Control Charts – Linear Failure Rate Distribution

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Abstract

The well-known Linear Failure Rate Distribution (LFRD) is considered. A process variate following LFRD is proposed in order to develop control charts for subgroup mean and subgroup range. In view of the limitations on LFRD the theoretical control limits are obtained through some approximations and the resulting control chart limits are worked out. Comparison with the control limits of similar variable control charts is also presented.

Keywords: Linear failure rate distribution, Control charts, Sub group mean and range.

1. Introduction

It is well-known that a control chart is a graphical device that detects variations in any variable quality characteristic of a product. Given a specified target value of the quality characteristic say μ_0 , production of the concerned product has to be so designed that the associated quality characteristic for the products should be ideally equal to μ_0 , if not , very close to μ_0 on its either side. That is, if the products are showing variations in the desirable quality, the variations must be within control in some admissible sense. That is, there should be two limits within which the allowable variations are supposed to fall. Whenever this happens, the production process is defined to be in control. Otherwise, it is out of control. Based on this principle it is necessary to think of the control limits on either side of the target value in such a way that under normal conditions the limits should include most of the observations. With this backdrop the well-known Shewhart control charts are developed under the assumption that the quality characteristic follows a normal distribution.

If $x_1, x_2, ..x_n$ is a collection of observations of size 'n' on a variable quality characteristic of a product, t_n is a statistic based on this sample, the control limits of Shewhart variable control chart are $E(t_n) \pm 3S.E.(t_n)$. Under repeated sampling of size 'n' at each time (say k times) the graph of the points (i, $t_n(i)$), i=1 to k, where $t_n(i)$ is the value of t_n based on n observations of the i^{th} sample, along with three lines parallel to horizontal axis at $E(t_n) - 3\,S.E.(t_n)$, $E(t_n)$ and $E(t_n) + 3\,S.E.(t_n)$ is called control chart for the statistic t_n. For instance, if t_n is $\bar{X}$, the graph is control chart for mean, if t_n is range the graph is control chart for range and so on. Assuming normality of the quality data we can get the control limits for $\bar{X}$ chart. But the limits for other charts like range, standard deviation if derived on the above principle may not be acceptable because of the fact that the distribution of t_n may not be normal. Even if asymptotic normality of t_n is made use of, it is valid only in large samples. However, in quality control studies data is always in small samples only. Therefore if the population is not normal there is a need to develop a separate procedure for the construction of control limits.

Skewed distributions to develop control charts are considered by many authors. Edgemen (1989), Kantam and Sriram (2001), Kantam *et al.* (2006), Kantam and Priya (2010), Kantam and Srinivasa Rao (2010), Kantam and Priya (2011), Srinivasa Rao and Kantam (2012) and the references therein are a few contributions in this direction. Besides these works, many researchers have been working on the theory of control charts for skewed as well as symmetrically distributed data that include. Amin and Miller (1993), Costa (1995), Costa (1996), Amin and Widmaier (1999) , Wu *et al.* (2002), Kan and Yazici (2006), Gob *et al.* (2006), Mahadik and Shirke (2007), Zhang *et al.* (2011), Derya and Canan (2012). In this paper we consider the well-known linear failure rate distribution (LFRD) - a skewed distribution and attempt to develop control limits for a variable quality characteristic assumed to follow LFRD.

The density function, cumulative distribution function, hazard or failure rate function of LFRD are

$$f(x) = (a + bx)e^{-\left(ax+\frac{bx^2}{2}\right)};\ x > 0, a > 0, b > 0, \tag{1.1}$$

$$F(x) = 1 - e^{-\left(ax+\frac{bx^2}{2}\right)}; x > 0, a > 0, b > 0, \tag{1.2}$$

$$h(x) = a + bx. \tag{1.3}$$

Ananda Sen (2005) gave a detailed review along with the distributional characteristics and inferential aspects of LFRD. Some basic features of LFRD are as follows:

Mean:

$$\mu = \sqrt{\frac{2\pi}{b}}\, e^{a^2/2b}\left(1 - \Phi(a/\sqrt{b})\right), \tag{1.4}$$

where $\Phi(.)$ denotes the cumulative distribution function of a standard normal variate.

Variance:

$$\sigma^2 = \frac{2}{b}(1 - a\mu) - \mu^2, \tag{1.5}$$

Mode:

$$M = \left(\sqrt{\frac{1}{b}} - \frac{a}{b}\right) I(a^2 < b), \tag{1.6}$$

where I(.) denotes indicator function.

100 p[th] Percentile:

$$F^{-1}(p) = \sqrt{\left(\frac{a}{b}\right)^2 - \frac{2\log(1-p)}{b}} - \frac{a}{b}, \tag{1.7}$$

and hence median is

$$M_d = \sqrt{\left(\frac{a}{b}\right)^2 - \frac{2\log(0.5)}{b}} - \frac{a}{b}, \tag{1.8}$$

The sampling distribution of mean and range of a random sample of size 'n' drawn from LFRD are not in analytical form thereby resulting in lack of exact percentiles of these sampling distributions analytically. Hence we have to try for approximate control limits/ corrected control limits if acceptable. We have addressed this problem in two different approaches.

(i) Fixing LFRD as a suitable model for a quality data and trying for approximate quality control constants for the data.

(ii) Approximating LFRD by a reasonable and admissible model for which exact quality control constants are available and making use of them for LFRD data.

For approach (i) we borrowed the results of Chan and Cui (2003). For approach (ii) we made use of the results in Kantam and Sriram (2001). The rest of the paper is organized as follows. The summary of Chan and Cui (2003) and its adoption to LFRD are given in Section 2. The content of Kantam and Sriram (2001) and its adoption to LFRD, the performance of the LFRD based control charts by the above two approaches are given in Section 3, followed by an example in Section 4.

2. LFRD Based Control Charts: Approach-I

(a) Principle of Skewness Corrected Control Chart (Chan and Cui, 2003)

Let X be a standardized random variable with mean 0, standard deviation 1, coefficient of skewness k_3. Let $x_1, x_2, .. x_n$ be a sample from the distribution of a process variate with mean μ and standard deviation σ. We know that when the process parameters are unknown the Shewhart limits are given by

Shewhart $\bar{X}$ Chart:
$UCL_{\bar{X}} = \bar{\bar{X}} + A_2\bar{R}, CL_{\bar{X}} = \bar{\bar{X}}, LCL_{\bar{X}} = \bar{\bar{X}} - A_2\bar{R},$

Shewhart R Chart:
$UCL_R = D_4\bar{R}, CL_R = \bar{R}, LCL_R = D_3\bar{R}.$
where the constants A_2, D_3, D_4 are available for specified sub group sizes from any standard text book on statistical quality control.

The control limits and the central line for a skewness corrected (SC) control chart for $\bar{X}$ chart are

$$SC\ \bar{X}\ Chart: \begin{cases} UCL_{\bar{X}} = \bar{\bar{X}} + \left(3 + \frac{4k_3/(3\sqrt{n})}{1+0.2k_3^2/n}\right)\frac{\bar{R}}{d_2^*\sqrt{n}} \equiv \bar{\bar{X}} + A_U^*\bar{R}, \\ CL_{\bar{X}} = \bar{\bar{X}}, \\ LCL_{\bar{X}} = \bar{\bar{X}} + \left(-3 + \frac{4k_3/(3\sqrt{n})}{1+0.2k_3^2/n}\right)\frac{\bar{R}}{d_2^*\sqrt{n}} \equiv \bar{\bar{X}} - A_L^*\bar{R}. \end{cases} \tag{2.1}$$

where the constant d_2^* is specially developed and is available in Chan and Cui (2003). The results of SC method control limits are tabulated for n= 2 (1) 5, 7, 10 in Chan and Cui (2003) and these are reproduced here in Table 2.1 for our adoption to LFRD corresponding to a list of selected values of population coefficient of skewness.

Table 2.1: SC $\bar{X}$ - chart constants A_U^* and A_L^*

	***n*=2**		***n*=3**		***n*=4**		***n*=5**		***n*=7**		***n*=10**	
k_3	A_U^*	A_L^*	A_U^*	A_L^*	A_U^*	A_L^*	A_U^*	A_L^*	A_U^*	A_L^*	A_U^*	A_L^*
0.0	1.88	1.88	1.03	1.03	0.73	0.73	0.58	0.58	0.42	0.42	0.31	0.31
0.4	2.14	1.67	1.13	0.92	0.82	0.69	0.63	0.53	0.45	0.39	0.33	0.29
0.8	2.37	1.47	1.25	0.84	0.87	0.61	0.68	0.50	0.48	0.37	0.35	0.28
1.2	2.61	1.32	1.37	0.77	0.95	0.57	0.74	0.46	0.52	0.35	0.37	0.26
1.6	2.83	1.22	1.49	0.72	1.03	0.54	0.79	0.44	0.56	0.33	0.39	0.25
2.0	3.02	1.15	1.60	0.68	1.10	0.51	0.85	0.42	0.59	0.32	0.42	0.25
2.4	3.19	1.12	1.69	0.65	1.18	0.49	0.91	0.40	0.63	0.30	0.44	0.23
2.8	3.32	1.13	1.78	0.64	1.24	0.47	0.95	0.39	0.66	0.29	0.46	0.22
3.2	3.45	1.16	1.86	0.64	1.29	0.47	1.00	0.38	0.69	0.29	0.48	0.22
3.6	3.52	1.20	1.92	0.65	1.34	0.47	1.04	0.37	0.72	0.28	0.50	0.21
4.0	3.59	1.52	1.97	0.66	1.39	0.47	1.07	0.37	0.75	0.27	0.51	0.21

If the value of k_3 for our specified model is not one of those in the above table it is suggested to take the nearest tabulated value of k_3 or to use interpolation.

Proceeding on similar lines the control limits for the skewness corrected range chart are given by

$$SC\ \bar{R}\ Chart:\begin{cases} UCL_R = \left[1 + (3 + d_4^*)\frac{d_3^*}{d_2^*}\right]\bar{R} \equiv D_4^*\bar{R}, \\ CL_{\bar{X}} = \bar{R}, \\ LCL_{\bar{X}} = \left[1 + (-3 + d_4^*)\frac{d_3^*}{d_2^*}\right]^+ \bar{R} \equiv D_3^*\bar{R}\,. \end{cases} \tag{2.2}$$

where d_2^*, d_3^*, d_4^* are control chart constants specially constructed taking into consideration the nonnormality of the model. For ready reference the constants for SC range chart are also reproduced here from Chan and Cui (2003) in Table 2.2.

Table 2.2: SC *R* - chart constants D_4^* and D_3^*

	***n*=2**		***n*=3**		***n*=4**		***n*=5**		***n*=7**		***n*=10**	
$\boldsymbol{k_3}$	$\boldsymbol{D_4^*}$	$\boldsymbol{D_3^*}$	$\boldsymbol{D_4^*}$	$\boldsymbol{D_3^*}$	$\boldsymbol{D_4^*}$	$\boldsymbol{D_3^*}$	$\boldsymbol{D_4^*}$	$\boldsymbol{D_3^*}$	$\boldsymbol{D_4^*}$	$\boldsymbol{D_3^*}$	$\boldsymbol{D_4^*}$	$\boldsymbol{D_3^*}$
0.0	4.12	0.00	2.93	0.00	2.53	0.00	2.30	0.10	2.06	0.24	1.88	0.35
0.4	4.21	0.00	3.06	0.00	2.69	0.01	2.40	0.14	2.16	0.27	1.98	0.38
0.8	4.41	0.00	3.28	0.00	2.85	0.07	2.61	0.17	2.36	0.29	2.17	0.39
1.2	4.70	0.00	3.58	0.00	3.13	0.09	2.88	0.17	2.61	0.28	2.41	0.37
1.6	5.03	0.00	3.90	0.00	3.44	0.07	3.17	0.15	2.88	0.26	2.65	0.34
2.0	5.32	0.00	4.20	0.00	3.71	0.03	3.44	0.11	3.13	0.21	2.90	0.28
2.4	5.60	0.00	4.46	0.00	3.97	0.00	3.69	0.06	3.37	0.16	3.11	0.24
2.8	5.85	0.00	4.71	0.00	4.21	0.00	3.92	0.05	3.58	0.11	3.31	0.19
3.2	6.09	0.00	4.93	0.00	4.42	0.00	4.13	0.00	3.78	0.00	3.50	0.14
3.6	6.27	0.00	5.12	0.00	4.61	0.00	4.31	0.00	3.96	0.00	3.67	0.09
4.0	6.44	0.00	5.30	0.00	4.79	0.00	4.48	0.00	4.11	0.00	3.81	0.04

If the distribution under consideration is a skewed one, its coefficient of skewness say k_3 is first evaluated by any standard formula. Particular to the subgroup size where a control chart for mean is needed, we identify the control limits A_L^*, A_U^* from the bivariate Table 2.1 with the help of linear interpolation if necessary. The pair (A_L^*, A_U^*) so selected when used in the formula (2.1) would give the control limits of the mean chart based on SC method.

(b) Adoption of Chan and Cui (2003) to Control Charts Based on LFRD Variate

We know that the LFRD is a skewed distribution. Here we chose the Bowley's, Kelly's formulae for finding coefficient of skewness which are respectively given by $k_{3(B)} = \frac{Q_3-2Q_2+Q_1}{Q_3-Q_1}$, $k_{3(K)} = \frac{P_{90}-2P_{50}+P_{10}}{P_{90}-P_{10}}$, where Q_i(i=1, 2, 3) is the i^{th} quartile and P_i (i= 10, 50, 90) is the percentile of the LFRD.

We fix the LFRD parameters for developing control chart constants as a=3, b=25. The Bowley's and Kelly's coefficients of skewness for LFRD are 0.117 and 0.2314 respectively. It can be seen from Table 2.1, that the values of our coefficients of skewness are not figured in Table 2.1. Accordingly, as per the suggestion in Chan and Cui (2003) we have resorted to interpolation in order to get the values of A_L^*, A_U^*, D_3^*, D_4^* corresponding to the k_3 values under discussion.

As a result of our interpolation technique the following are the values of A_L^* ,A_U^* and D_3^* , D_4^* corresponding to specified choices of n and k_3. These are given in the following Tables 2.3 and 2.4 respectively.

Table 2.3: SC $\bar{X}$-Chart Constants

Coefficient of Skewness (B) = 0.117			Coefficient of Skewness (K) = 0.2314		
n	A_U^*	A_L^*	***n***	A_U^*	A_L^*
2	1.9560	1.8185	2	2.0304	1.7585
3	1.0592	0.9978	3	1.0878	0.9663
4	0.7563	0.7183	4	0.7820	0.7068
5	0.5946	0.5653	5	0.6089	0.5510
7	0.4287	0.4112	7	0.4373	0.4026
10	0.3158	0.3041	10	0.3215	0.2984

Table 2.4: SC $\bar{R}$-Chart Constants

Coefficient of Skewness (B) = 0.117			Coefficient of Skewness (K) = 0.2314		
n	D_4^*	D_3^*	***n***	A_U^*	A_L^*
2	4.1463	0	2	4.1720	0
3	2.9680	0	3	3.0052	0
4	2.5768	0	4	2.6225	0
5	2.3292	0.1117	5	2.3578	0.1213
7	2.0892	0.2487	7	2.1178	0.2573
10	1.9092	0.3587	10	1.9378	0.3673

3. LFRD Based Control Charts: Approach-II

(a) Exact Control Limits for Gamma Variate: Kantam and Sriram (2001)

The probability density function of the gamma distribution having shape parameter 2 and scale parameter σ is given by

$$g(x) = (x/\sigma^2)\exp(-x/\sigma)\,,\ \sigma > 0, x > 0. \tag{3.1}$$

If $x_1, x_2, .. x_n$ is a random sample of size n from a gamma distribution with shape parameter 2 and scale parameter σ, it is known that $\bar{x}/2$ is the maximum likelihood estimator of σ. It is the UMVUE of σ as well. The sampling distribution of $n\bar{x}/\sigma$ is gamma with shape parameter $2n$. This fact can be used to get equitailed 99.73% probability limits of $\bar{x}$ analogous to the corresponding 3σ limits of an $\bar{X}$chart in the case of normal distributions. Let L_1 and L_2 be these two equitailed 99.73% percentiles of the gamma distribution with shape parameter $2n$. Then,

$$\left.\begin{aligned} P\left\{L_1 \le \frac{n\bar{x}}{\sigma} \le L_3\right\} = 0.9973 \\ P\left\{\frac{n\bar{x}}{\sigma} < L_1\right\} = 0.00135 \\ P\left\{\frac{n\bar{x}}{\sigma} < L_2\right\} = 0.99865 \end{aligned}\right\} \tag{3.2}$$

L_1 and L_2 depend on the sample size n. Using tables of the incomplete gamma function to find these two values for a given n, the following probability statement can be made:

$$P\{L_1\,\sigma/n < \bar{x} < L_2\,\sigma/n\} = 0.9973 \tag{3.3}$$

That is, in repeated sampling say, (*k* times) of size *n* each time with the arithmetic mean of the i^{th} sample as $\bar{x}_i$, *i*=1,2,….,*k*, a plot of the serial number of the sample against its corresponding arithmetic mean, provides a graph of the control chart for averages. The unknown parameter σ is estimated by its maximum likelihood estimator, $\bar{x}/2$. Over repeated sampling (subgrouping), σ is estimated by $\bar{\bar{x}}/2$, where $\bar{\bar{x}}$ is the grand mean of all the subgroups. Then estimated version of (3.3) becomes,

$$P\{M_1\bar{\bar{X}} < \bar{X} < M_2\bar{\bar{X}}\} = 0.9973,$$

where $M_i = L_i/2n$; $i = 1, 2$; which are given in Table 3.1. These can be used to get the control chart limits.

To develop the control chart for the ranges, the percentile points of the sampling distribution of the sample range are needed. In the case of the gamma distribution, these percentile points of the sample range are not available in a published form till 2001. These are tabulated in Kantam and Sriram (2001) with the following methodology. If W is the range of a sample of size n from a continuous probability model with *f*(.) and *F*(.) as the probability density function and cumulative distribution function respectively, it is known that the cumulative distribution of W is given as:

$$G(w) = n\int_{-\infty}^{+\infty} f(x)[F(x+w) - F(x)]^{n-1}dx. \tag{3.4}$$

Replacing *f*(.) and *F*(.) in equation (3.4) with the corresponding gamma distribution function with shape parameter 2, the following expression can be obtained:

$$G(w) = \sum_{j=0}^{n-1}\left\{(-1)^j\binom{n-1}{j}w^j e^{-jw}\left[\sum_{k=0}^{(n-j-1)}(-1)^k e^{-kw}\binom{n-j-1}{k}\right]\right.$$
$$\left.\left[\sum_{i=0}^{n-j-1}\binom{n-j-1}{i}\left((i+1)!/n^{i+1}\right)\right]\right\} \tag{3.5}$$

From this equation, the values of *G(w)* have been tabulated for *n* =2(1)10, *w*= 0.0 (0.05) 14.55.

In order to get the control limits for range chart we have to find two constants c_1, c_2 such that

$$P\{c_1 < R < c_2\} = 0.9973 \tag{3.6}$$

If '*w*' is the sample range in a standard gamma $\left(w = Z_{(n)} - Z_{(1)}\right)$ we know that $R = \sigma w$. Hence equation (3.6) becomes

$$\mathbf{P}\{\boldsymbol{c_1} < \sigma w < \boldsymbol{c_2}\} = \mathbf{0.9973}$$

$$\boldsymbol{P}\left\{\frac{\boldsymbol{c_1}}{\boldsymbol{\sigma}} < w < \frac{\boldsymbol{c_2}}{\boldsymbol{\sigma}}\right\} = \mathbf{0.9973}$$

i.e., c_1/σ, c_2/σ would respectively be the 0.00135, 0.99865 percentiles of the distribution of sample range, which from tabulated values of (3.5) are available in Kantam and Sriram (2001).

Let $\boldsymbol{c_1 = w_{0.00135}\sigma, c_2 = w_{0.99865}\sigma}$

i.e., $\boldsymbol{c_1 = \frac{w_{0.00135}R}{\alpha_{(n)}-\alpha_{(1)}}, c_2 = \frac{w_{0.99865}R}{\alpha_{(n)}-\alpha_{(1)}}}$

i.e., $c_1=M_3R$, $c_2=M_4R$

where $\boldsymbol{M_3 = \frac{w_{0.00135}}{\alpha_{(n)}-\alpha_{(1)}}, M_4 = \frac{w_{0.99865}}{\alpha_{(n)}-\alpha_{(1)}}}$

The constants M_3, M_4 depend only on n, the mathematical model of standard gamma distribution and its moments of extreme sample order statistics. These can be calculated a priori outside the data set. Over repeated sub grouping R is replaced by the mean of ranges$\bar{R}$. Thus the control limits of range chart would be $M_3\bar{R}, M_4\bar{R}$ where M_3, M_4 are given in Table 3.1.

Table -3.1: Control Chart Constants

	$\bar{X}$ **- Chart Constants**		**Range Chart Constants**	
n	M_1	M_2	M_3	M_4
2	0.1111	3.1710	0.00360	5.48361
3	0.1928	2.6733	0.03262	4.05515
4	0.2561	2.3972	0.08986	3.51024
5	0.3065	2.2179	0.14083	3.21090
6	0.3486	2.0908	0.18418	3.01330
7	0.3818	1.9945	0.22178	2.88586
8	0.4109	1.9185	0.25293	2.78720
9	0.4239	1.8046	0.27945	2.70410
10	0.4583	1.8057	0.30226	2.64058

(b) Adoption of Kantam and Sriram (2001) to LFRD

As mentioned in Section 1, we try to mitigate the problem of non availability of exact control chart constants for LFRD by using exact control chart constants of gamma distribution with LFRD data. That is, data will be generated form LFRD and exact limits of gamma model will be used to develop control chart limits. In this direction we have attempted the following simulation methodology.

Random samples of size n= 2,3,4,5,7,10 are generated from LFRD a=3, b=25. For each sample, the mean and range are calculated, the grand mean and the mean of the ranges are also computed. Using the constants A_L^* , A_U^* of Table 2.3; M_1, M_2 of Table 3.1, the control limits of $\bar{X}$ – chart for LFRD data are calculated. The proportion of sub-group means that fall within pair of control limits out of 10000 runs is noted down. This proportion is named as coverage probability of the respective pair of control limits. Similarly using the constants D_3^* , D_4^* of Table 2.4 and the constants M_3, M_4 of Table 3.1 two pairs of control limits for range chart need to be calculated. The proportion of subgroup ranges that fall within each pair of control limits out of 10,000 runs is noted down. These proportions are named as coverage probabilities of respective control limits for range chart. A consolidated table of the control limits and the corresponding coverage probabilities for the various pairs of control limits are presented separately for $\bar{X}$ chart and Range chart in Table 3.2 and Table 3.3 respectively.

From Table 3.2 we see that $\bar{X}$ chart based on skewness correction control limits using Kelly's coefficient of skewness seems to be preferable with respect to coverage probabilities. On the other hand, Table 3.3 shows that adoption of exact control chart constants of gamma distribution for LFRD data is rated as preferable. We therefore suggest skewness corrected constants for $\bar{X}$ chart and exact constants of gamma distribution for range chart of LFRD data.

Table 3.2: Control Limits for $\bar{X}$ - Chart

n	Skewness Correction Limits						Adoption of Exact Gamma Distribution Constants		
	Bowley's Coefficient of Skewness			Kelly's Coefficient of Skewness					
	LCL	UCL	Coverage Probabilities	LCL	UCL	Coverage Probabilities	LCL	UCL	Coverage Probabilities
2	0	0.4140	0.9948	0	0.4236	0.9956	0.0180	0.5158	0.9914
3	0	0.3429	0.9967	0	0.3484	0.9979	0.0270	0.5117	0.9937
4	0	0.3394	0.9964	0	0.3454	0.9972	0.0416	0.3899	0.9968
5	0.0144	0.3185	0.9990	0.0182	0.3223	0.9990	0.0498	0.3608	0.9965
7	0.0385	0.2921	0.9971	0.0411	0.2947	0.9985	0.0621	0.3245	0.9986
10	0.0584	0.3940	1.0000	0.0604	0.2728	0.9980	0.0745	0.2938	0.9960

Table 3.3: Control Limits for Range Chart

n	Skewness Correction Limits						Adoption of Exact Gamma Distribution Constants		
	Bowley's Coefficient of Skewness			Kelly's Coefficient of Skewness					
	LCL	UCL	Coverage Probabilities	LCL	UCL	Coverage Probabilities	LCL	UCL	Coverage Probabilities
2	0	0.5328	0.9978	0	0.5361	0.9978	0.0005	0.8920	0.9958
3	0	0.5682	0.9958	0	0.5753	0.9979	0.0062	0.7763	0.9988
4	0	0.6021	0.9984	0	0.6127	0.9984	0.0209	0.8202	0.9996
5	0.0292	0.6106	0.9970	0.0318	0.6181	0.9965	0.0369	0.8417	0.9985
7	0.0750	0.6307	0.9957	0.0776	0.6393	0.9950	0.0669	0.8712	0.9993
10	0.1229	0.6542	0.9940	0.1258	0.6640	0.9940	0.1035	0.9048	0.9990

4. Example

We illustrate the use of the SC method with a numerical example. The following data describes the thickness of paint on refrigerators for five refrigerators from each shift. The table 4.1 shows the data for 20 subgroups of size n=5 from a process that is known to be in control.

Table 4.1: The thickness of Paint on Refrigerators for Five Refrigerators from Each Shift

S. No	1	2	3	4	5
1	2.7	2.3	2.6	2.4	2.7
2	2.6	2.4	2.6	2.3	2.8
3	2.3	2.3	2.4	2.5	2.4
4	2.8	2.3	2.4	2.6	2.7
5	2.6	2.5	2.6	2.1	2.8
6	2.2	2.3	2.7	2.2	2.6
7	2.2	2.6	2.4	2.0	2.3
8	2.8	2.6	2.6	2.7	2.5
9	2.4	2.8	2.4	2.2	2.3
10	2.6	2.3	2.0	2.5	2.4
11	3.1	3.0	3.5	2.8	3.0
12	2.4	2.8	2.2	2.9	2.5
13	2.1	3.2	2.5	2.6	2.8
14	2.2	2.8	2.1	2.2	2.4
15	2.4	3.0	2.5	2.5	2.0
16	3.1	2.6	2.6	2.8	2.1
17	2.9	2.4	2.9	1.3	1.8
18	1.9	1.6	2.6	3.3	3.3
19	2.3	2.6	2.7	2.8	3.2
20	1.8	2.8	2.3	2.0	2.9

The skewness for this data is -0.16846. We have used the interpolated control chart constant values based on skewness correction method from Tables 2.1 and 2.2, for the construction of Mean and Range charts. The exact limits of gamma are calculated using the constants of Table 3.1. The coverage probabilities and average run length values for the mean and range charts are given in the following table.

Mean Chart:

SC Limits			Gamma Limits			Shewart Limits		
LCL	UCL	CP (ARL)	LCL	UCL	CP(ARL)	LCL	UCL	CP(ARL)
2.051186	2.944386	0.95 (11th Observation)	2.277995	4.221783	0.95 (17th Observation)	2.06971	2.95829	0.55 (17th Observation)

Range Chart:

SC Limits			Gamma Limits			Shewart Limits		
LCL	UCL	CP	LCL	UCL	CP	LCL	UCL	CP
0.089971	1.803429	1	0.108439	2.472393	1	0	1.62778	0.95

From the above tables, we observe that though the skewness corrected limits and gamma limits are having the same coverage probabilities, skewness corrected limits alert faster when compared to others in the case of mean chart. But for Range chart, both the skewness corrected limits and gamma limits are having same coverage probabilities. However, in either case Shewart limits have recorded a less coverage probability and a delayed alert of out of control indicating that mechanical usage of Shewart limits for non-normal data results in admissible conclusions.

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