Stress Affection of Two Scale Truncated Generalized Logistic Parameters with Progressive Censoring

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Abstract

This paper deals with non-Bayesian estimation problem of constant-stress Accelerated Life Tests (ALTs) when the lifetime of the items follow truncated Generalized Logistic Distribution (GLD). Some considerations on inference based on the use of asymptotically normality of the ML estimators are presented considering the stress effects on the two scale parameters of the truncated GLD with a k-level constant-stress ALT under progressive type-I censored grouped data. The EM algorithm method is used to obtain the estimators of the unknown parameters. In addition, estimator of the two scale parameters, reliability function under usual conditions and Fisher information matrix of the estimators are given. Finally, we present a Simulation Study to illustrate the proposed procedure.

Keywords and phrases: Truncated Generalized Logistic Distribution; Constant-stress Accelerated Life Test; EM algorithm method; Fisher Information Matrix; Progressive Type-I Censored Grouped Data.

1. Introduction

With today's highly reliable components, we are often unable to obtain a reasonable amount of test data under normal use condition. For this reason, ALT is the reasonable procedure to be applied. ALT is used to determine the reliability of a product in a short period of time by accelerating the use environment. Nelson (1990) has indicated that the stress can be applied in various ways, commonly used methods are step stress and constant stress ALT. In real life, many products such as, insulations, bearing, electronic components, semiconductors, and microelectronics run at a constant stress (i.e., run under constant temperature, voltage, load, and etc). CSALT is the most comely adopted ALT so that it is the easiest way to run and estimate reliability information for high reliability and long lifetime products under usual conditions. In CSALT, the stress is kept at a constant level of stress throughout the life test, i.e., each unit is run at a constant high stress level until the occurrence of failure or the observation is censored. Since, CSALT is a widely used in assessing the reliability of products. There is abundant literature on how to design CSALT. They differ in the assumed lifetime distribution, censoring scheme, and test condition. For example, Meeker and Nelson (1975) proposed statistically optimal plans for CSALT which involved only two stress levels. Meeker and Hahn (1977) considered optimal allocation of test units to accelerated stress conditions with the objective of minimizing the estimate of the product reliability under usual condition. Nelson (1990) reviewed statistically optimal and compromise plans for the single stress ALT planning

problem. Another approach adopted by Yang (1994) and Yang and Jin (1994) to overcome the problem of Meeker and Nelson (1975) was to add test constraints involving expected minimum number of failures to be observed at the middle stress levels. Tang, *et al.* (2002) presented two alternative ways of CSALT planning with three stress levels that optimize the sample and the sample locations. Tang and Xu (2005) considered a framework with two conflicting objectives in the planning of ALT, meeting a desired level of statistical precision for an estimate of interest, and meeting a cost target for conducting the test. Ding *et al.* (2010) considered the design of ALT sampling plans under type-I progressive interval censoring with random removals and assumed the lifetime of products follows a Weibull distribution. Attia, *et al.* (2011a,b) obtained MLE and discussed optimum test plans of the Generalized Logistic (GL) parameters under both type-I and type-II censoring data. It is assumed that the stress affects only the scale GL parameter. The term Generalized Logistic (GL) distribution is used as the name for several different families of probability distributions [see Johnson, *et al.* (1995)]. The main feature of the GL distribution is that new parameters were introduced to control both location and scale. It allows for a greater degree of flexibility and it is expected to be useful in many more practical situations Nadarajah and Kotz (2005).

Truncated distributions arise when sample selection is not possible in some sub-region of the sample space. The GLD is considered inappropriate for modeling lifetime data because left hand side of its distribution extends to negative infinity, and this could conceivably result in modeling negative times-to-failure. This has necessitated the use of truncated GLD truncated at point zero for modeling lifetime data.

This paper, considers CSALT under truncated GLD using progressive type-I censored grouped data. In addition to this introductory section this paper includes other Sections too. In Section 2, the model and assumptions are described. Section 3 presents the estimation method. EM algorithm method for obtaining the estimators of the unknown parameters are described in Section 4. In Section 5 the method developed has been illustrated using the data simulated from the proposed models.

2. Model and Assumptions

The generalized logistic distribution has been addressed by Molenberghs and Verbeke (2011), which had the following cumulative distribution function

$$F(x) = 1 - \left(1 + \frac{\alpha_2}{\theta} e^{\alpha_1 x}\right)^{-\theta}, \quad -\infty < x < \infty,$$

where θ is a positive shape parameter, and α_1, α_2 are positive scale parameters. In this paper, we use the truncated GLD (truncated at point zero for modeling lifetime data), Thus the cumulative distribution function is given by the following form

$$F(x, \alpha_1, \alpha_2) = 1 - \left(1 + \frac{\alpha_2}{\theta}\right)^{\theta} \left(1 + \frac{\alpha_2}{\theta} e^{\alpha_1 x}\right)^{-(\theta+1)}, \quad x, \theta, \alpha_q > 0, q = 1,2$$

We assume the test is done at high stresses v_j, $j = 1, \ldots, k$ where $v_1 < v_2 < \ldots < v_k$ and v_u be the design stress; that is the stress level under usual conditions, where $v_u < v_j$, $j = 1, 2, \ldots, k$. For the testing procedure, a total of N units are divided into n_1, $n_2, \ldots, n_k$ units where $\sum_{j=1}^{k} n_j = N$. Each n_j, $j = 1, 2, \ldots, k$ units in the experiment are run at a pre-specified constant stress v_j, $j = 1, 2, \ldots, k$. The life of a test unit at each

stress is described by the truncated three-parameter GLD. Thus, the cumulative distribution function of the lifetime of a test unit under k-level step-stress test is

$$F(x_{ij}, \alpha_{1j}, \alpha_{2j}) = 1 - \left(1 + \frac{\alpha_{2j}}{\theta}\right)^{\theta} \left(1 + \frac{\alpha_{2j}}{\theta} e^{\alpha_{1j} x_{ij}}\right)^{-(\theta+1)}, \quad x_{ij}, \theta, \alpha_{qj} > 0, q = 1,2$$

The scale parameters α_{qj}, $j = 1, 2, \ldots, k$, $q = 1, 2$ at the stress level v_j, $j = 1, 2, \ldots, k$ of a test unit is a power law function of stress [see, Mann, Schafer and Singurwalla(1974)], given by

$$\alpha_{qj} = c_q \, v_j^p,$$

where $c_{q,}\, p$ constants to be estimated. Progressively censored sampling is an important method for lifetime studies. This method allows the experimenter to save time and cost and it is useful when the items being tested are very expensive. Let us consider there are k-stage constant stress ALT scheme with progressive Type-I interval censoring. In this case, the test will be carrying out as follows: n units are simultaneously placed on a life test at stress level v_1, and run until the time x_1. At this time, the number of failed units r_1 are counted and R_1 surviving units are removed from the test; starting from the time x_1, the $(n - r_1 - R_1)$ non-removed surviving units are put to a stress level v_2, $v_1, < v_2$, and run until the time x_2. At this time, the number of failed units r_2 are counted and R_2 surviving units are removed from the test. Starting from the time x_2, the $(n - r_1 - R_1 - r_2 - R_2)$ non-removed surviving units are put to a stress level v_3, $v_2, < v_3$ and run until the time x_3. At this time, the number of failed units r_3 are counted and R_3 surviving units are removed from the test, and so on. At the time x_k, the number of failed units r_k are counted and the remaining surviving units $R_k = n - \sum_{j=1}^{k} r_j - \sum_{j=1}^{k-1} R_j$ are all removed, thereby terminating the test, [Balakrishnan and Aggarwala(2000)].

3. Estimation Method

Suppose a type-I progressively censored grouped sample is collected as described in Section 2 and by assuming that the experiment is done under k-stage constant stress level, the number of failed units r_j, $j = 1, 2, \ldots, k$ which are observed while the testing in the interval (x_{j-1}, x_j) at stress v_j, $j = 1, 2, \ldots, k$ are random variables with the fact that $r_j | r_{j-1}, \ldots, r_1 \sim$ Binomial $[m_j, F_j(x)]$, where $m_j = N - \sum_{i=1}^{j-1} r_i - \sum_{i=1}^{j-1} R_i$ is the number of non-removal surviving units at the beginning of the jth stage, and $F_j(x) = \frac{F(x_j) - F(x_{j-1})}{1 - F(x_{j-1})}$. Therefore, the cdf of the lifetime distribution at the stress level v_j, $j = 1, 2, \ldots, k$ and in the inspection interval (x_{j-1}, x_j) will take the following form

$$F(x_{ij}, \alpha_{1j}, \alpha_{2j}) = 1 - \left(1 + \frac{\alpha_{2j}}{\theta}\right)^{\theta} \left(1 + \frac{\alpha_{2j}}{\theta} e^{\alpha_{1j} x_{ij}}\right)^{-(\theta+1)}, \quad x_{ij}, \theta, \alpha_{qj} > 0, q = 1,2$$

where α_{qj} is defined by Eq.(1). Therefore, the likelihood function is then

$$L = \prod_{j=1}^{k} [F(x_j) - F(x_{j-1})]^{r_j} \cdot [1 - F(x_j)]^{R_j}. \tag{2}$$

In the special case where the intervals are of equal length, so monitoring and censoring occurs periodically, say $x_{j=} j \cdot x$ [Aggarwala (2001)]. Therefore, the likelihood function (2) can be re-written as the following form

$$L = \prod_{j=1}^{k}[F(j\,x) - F((j-1)x)]^{r_j} \cdot [1 - F(j\,x)]^{R_j}. \qquad (3)$$

Substituting for α_{qj} the expression from (1) involving c_1, c_2, p the likelihood function can be written as

$$L = \prod_{j=1}^{k}\left(1 + \frac{c_2\,v_j^p}{\theta}\right)^{\theta R_j}\left(1 + \frac{c_2\,v_j^p}{\theta}\,e^{j\,c_1\,v_j^p\,x}\right)^{-\theta R_j}\left[\left(1 + \frac{c_2\,v_j^p}{\theta}\right)^{\theta}\left(1 + \frac{c_2\,v_j^p}{\theta}\,e^{(j-1)\,c_1\,v_j^p\,x}\right)^{-\theta} - \right.$$

$$\left. \left(1 + \frac{c_2\,v_j^p}{\theta}\right)^{\theta}\left(1 + \frac{c_2\,v_j^p}{\theta}\,e^{j\,c_1\,v_j^p\,x}\right)^{-\theta}\right]^{r_j}. \qquad (4)$$

Therefore, the log-likelihood function is
$$\ln L = \sum_{j=1}^{k}\{R_j\ln A_j(\emptyset) + (r_j + R_j)\ln A_j(\varepsilon) + r_j\ln[A_{j-1}(\emptyset) - A_j(\emptyset)]\}, \qquad (5)$$
where
$$\underline{\emptyset} = (c_1, c_2, p, \theta),\ \underline{\varepsilon} = (c_2, p, \theta),\ \underline{\omega} = (c_1, p),$$

$$A_j(\varepsilon) = \left(1 + \frac{c_2\,v_j^p}{\theta}\right)^{\theta}\ A_j(\emptyset) = \left(1 + B_j(\emptyset)\right)^{-\theta},\ B_j(\emptyset) = \frac{c_2\,v_j^p}{\theta}\,e^{j\,c_1\,v_j^p\,x},$$

$$A_{j-1}(\emptyset) = \left(1 + B_{j-1}(\emptyset)\right)^{-\theta},\ B_{j-1}(\emptyset) = \frac{c_2\,v_j^p}{\theta}\,e^{(j-1)\,c_1\,v_j^p\,x}.$$

From $\frac{\partial \ln L}{\partial c_1} = 0, \frac{\partial \ln L}{\partial c_2} = 0,\ \frac{\partial \ln L}{\partial p} = 0,$ and $\frac{\partial \ln L}{\partial \theta} = 0$ we find the maximum likelihood estimators $(\hat{c}_1, \hat{c}_2, \hat{p}, \hat{\theta})$ by solving the following four likelihood equations,

$$\sum_{j=1}^{k} j\,R_j\,B_j(\widehat{\emptyset})A_j^{\frac{1}{\hat{\theta}}}(\widehat{\emptyset}) = r_j\left(A_{j-1}(\widehat{\emptyset}) - A_j(\widehat{\emptyset})\right)^{-1}\left[j\,B_j(\widehat{\emptyset})A_j^{\frac{1}{\hat{\theta}}+1}(\widehat{\emptyset}) - (j-1)\,B_{j-1}(\widehat{\emptyset})A_{j-1}^{\frac{1}{\hat{\theta}}+1}(\widehat{\emptyset})\right] \qquad (6)$$

$$\sum_{j=1}^{k}\left\{(r_j + R_j)v_j^{\hat{p}}A_j^{\frac{-1}{\hat{\theta}}}(\hat{\varepsilon}) - \frac{\hat{\theta}R_j}{\hat{c}_2}\,B_j(\widehat{\emptyset})A_j^{\frac{1}{\hat{\theta}}}(\widehat{\emptyset})\right\} = \frac{r_j\left[B_j(\widehat{\emptyset})A_j^{\frac{1}{\hat{\theta}}+1}(\widehat{\emptyset}) - (j-1)\,B_{j-1}(\widehat{\emptyset})A_{j-1}^{\frac{1}{\hat{\theta}}+1}(\widehat{\emptyset})\right]}{\left(A_{j-1}(\widehat{\emptyset}) - A_j(\widehat{\emptyset})\right)} \qquad (7)$$

$$\sum_{j=1}^{k}\left\{\frac{R_j\,B_j(\widehat{\emptyset})\,A_j(\hat{\omega})}{A_j^{\frac{-1}{\hat{\theta}}}(\widehat{\emptyset})} - \frac{\hat{c}_2\,v_j^{\hat{p}}(r_j + R_j)}{\hat{\theta}A_j^{\frac{-1}{\hat{\theta}}}(\hat{\varepsilon})}\right\} = \frac{r_j\left[B_j(\widehat{\emptyset})\,A_j(\hat{\omega})A_j^{\frac{1}{\hat{\theta}}+1}(\widehat{\emptyset}) - (j-1)\,B_{j-1}(\widehat{\emptyset})\,A_{j-1}(\hat{\omega})A_{j-1}^{\frac{1}{\hat{\theta}}+1}(\widehat{\emptyset})\right]}{\left(A_{j-1}(\widehat{\emptyset}) - A_j(\widehat{\emptyset})\right)} \qquad (8)$$

$$\sum_{j=1}^{k}\left\{\left(r_j + R_j\right)\left[\ln\left(1 + \frac{\hat{c}_2 \, v_j^{\hat{p}}}{\hat{\theta}}\right) - \frac{\hat{c}_2 \, v_j^{\hat{p}}}{\hat{\theta} A_j^{\frac{1}{\hat{\theta}}}(\hat{\varepsilon})}\right] - R_j\left[\ln\left(1 + B_j(\hat{\emptyset})\right) - \right.$$

$$\left. B_j(\hat{\emptyset})A_j^{\frac{1}{\hat{\theta}}}(\hat{\emptyset})\right]\right\} = \frac{r_j\left[\left[A_j(\hat{\emptyset})\left[\ln\left(1 + B_j(\hat{\emptyset})\right) - B_j(\hat{\emptyset})A_j^{\frac{1}{\hat{\theta}}}(\hat{\emptyset})\right] - Q_{j-1}(\hat{\emptyset})\right]\right]}{\left(A_{j-1}(\hat{\emptyset}) - A_j(\hat{\emptyset})\right)} \tag{9}$$

where, $Q_{j-1}(\hat{\emptyset}) = A_{j-1}(\hat{\emptyset})\left[\ln\left(1 + B_{j-1}(\hat{\emptyset})\right) - B_{j-1}(\hat{\emptyset})A_{j-1}^{\frac{1}{\hat{\theta}}}(\hat{\emptyset})\right].$

To predict the value of the two scale parameters α_{1u} and α_{2u} under the usual condition stress v_u, the invariance property of MLE is used. Thus, the MLE of $\hat{\alpha}_{1u}$, $\hat{\alpha}_{2u}$ and $\hat{R}_u(x_0)$ at the lifetime x_0 under the usual condition stress v_u respectively, are given by the following equations

$$\hat{\alpha}_{1u} = \hat{c}_1 \, v_u^{\hat{p}}, \quad \hat{\alpha}_{2u} = \hat{c}_2 \, v_u^{\hat{p}}, \quad \hat{R}_u(x_0) = \left(1 + \frac{\hat{\alpha}_{2u}}{\hat{\theta}}\right)\left(1 + \frac{\hat{\alpha}_{1u}}{\hat{\theta}} \, e^{\,\hat{\alpha}_{1u}x_0}\right)^{-\hat{\theta}}$$

For inference on $\hat{\Phi} = \left(\hat{c}_1, \hat{c}_2, \hat{p}, \hat{\theta}\right)$ we usually use the asymptotical normality of the maximum likelihood estimators given by

$$\hat{\Phi} = \left(\hat{c}_1, \hat{c}_2, \hat{p}, \hat{\theta}\right) \sim N\left(\underline{\Phi}, I_0^{-1}\right),$$

Where

$$I_0 = \begin{pmatrix} I_{11} & I_{12} & I_{13} & I_{14} \\ I_{21} & I_{22} & I_{23} & I_{24} \\ I_{31} & I_{32} & I_{33} & I_{34} \\ I_{41} & I_{42} & I_{43} & I_{44} \end{pmatrix}$$

and

$$I_{11} = -\frac{\partial^2 \ln L(\Phi)}{\partial c_1^2} = \theta x^2 \sum_{j=1}^{k} v_j^{2p}\left\{j^2 \, R_j \, B_j(\emptyset)A_j^{\frac{1}{\theta}}(\emptyset)\left[1 - B_j(\emptyset)A_j^{\frac{1}{\theta}}(\emptyset)\right] + r_j\left[A_{j-1}(\emptyset) - \right.\right.$$

$$A_j(\emptyset)\right]^{-1}\left\{\left[j^2 B_j(\emptyset)A_j^{1+\frac{1}{\theta}}(\emptyset)\left[(\theta + 1)B_j(\emptyset)A_j^{\frac{1}{\theta}}(\emptyset) - 1\right] - (j - 1)^2 B_{j-1}(\emptyset)A_{j-1}^{1+\frac{1}{\theta}}(\emptyset)\left[(\theta + \right.\right.$$

$$1)B_{j-1}(\emptyset)A_{j-1}^{\frac{1}{\theta}}(\emptyset) - 1\right] + \theta\left[A_{j-1}(\emptyset) - A_j(\emptyset)\right]^{-1}\left[jB_j(\emptyset)A_j^{1+\frac{1}{\theta}}(\emptyset) - (j - \right.$$

$$1)B_{j-1}(\emptyset)A_{j-1}^{1+\frac{1}{\theta}}(\emptyset)\right]^2\right\}\right\},$$

$$I_{12} = -\frac{\partial^2 \ln L(\Phi)}{\partial c_1 \partial c_2} = \frac{\theta x}{c_2}\sum_{j=1}^{k} v_j^{2p}\left\{j \, R_j \, B_j(\emptyset)A_j^{\frac{1}{\theta}}(\emptyset)\left[1 - B_j(\emptyset)A_j^{\frac{1}{\theta}}(\emptyset)\right] + r_j\left[A_{j-1}(\emptyset) - \right.\right.$$

$$A_j(\emptyset)\right]^{-1}\left\{\left[jB_j(\emptyset)A_j^{1+\frac{1}{\theta}}(\emptyset)\left[(\theta + 1)B_j(\emptyset)A_j^{\frac{1}{\theta}}(\emptyset) - 1\right] - (j - 1)B_{j-1}(\emptyset)A_{j-1}^{1+\frac{1}{\theta}}(\emptyset)\left[(\theta + \right.\right.$$

$$1)B_{j-1}(\emptyset)A_{j-1}^{\frac{1}{\theta}}(\emptyset) - 1\Big]\Big] + \theta\big[A_{j-1}(\emptyset) - A_j(\emptyset)\big]^{-1}\Big[B_j(\emptyset)A_j^{1+\frac{1}{\theta}}(\emptyset) -$$

$$B_{j-1}(\emptyset)A_{j-1}^{1+\frac{1}{\theta}}(\emptyset)\Big]\Big[jB_j(\emptyset)A_j^{1+\frac{1}{\theta}}(\emptyset) - (j-1)B_{j-1}(\emptyset)A_{j-1}^{1+\frac{1}{\theta}}(\emptyset)\Big]\Big\}\Big\},$$

$$I_{13} = -\frac{\partial^2 \ln L(\Phi)}{\partial c_1 \partial p} = \theta x \sum_{j=1}^{k} v_j^p \ln v_j \Big\{ j\,R_j\,B_j(\emptyset)A_j(\omega)A_j^{\frac{1}{\theta}}(\emptyset)\Big[1 - B_j(\emptyset)A_j^{\frac{1}{\theta}}(\emptyset)\Big] +$$

$$r_j\big[A_{j-1}(\emptyset) - A_j(\emptyset)\big]^{-1}\Big\{\Big[jB_j(\emptyset)A_j(\omega)A_j^{1+\frac{1}{\theta}}(\emptyset)\Big[(\theta+1)B_j(\emptyset)A_j^{\frac{1}{\theta}}(\emptyset) - 1\Big] - (j-$$

$$1)B_{j-1}(\emptyset)A_{j-1}(\omega)A_{j-1}^{1+\frac{1}{\theta}}(\emptyset)\Big[(\theta+1)B_{j-1}(\emptyset)A_{j-1}^{\frac{1}{\theta}}(\emptyset) - 1\Big]\Big] + \theta\big[A_{j-1}(\emptyset) -$$

$$A_j(\emptyset)\big]^{-1}\Big[B_j(\emptyset)A_j(\omega)A_j^{1+\frac{1}{\theta}}(\emptyset) - B_{j-1}(\emptyset)A_{j-1}(\omega)A_{j-1}^{1+\frac{1}{\theta}}(\emptyset)\Big]\Big[jB_j(\emptyset)A_j^{1+\frac{1}{\theta}}(\emptyset) - (j-$$

$$1)B_{j-1}(\emptyset)A_{j-1}^{1+\frac{1}{\theta}}(\emptyset)\Big]\Big\}\Big\},$$

$$I_{14} = -\frac{\partial^2 \ln L(\Phi)}{\partial c_1 \partial \theta} = x \sum_{j=1}^{k} v_j^p \Big\{ j\,R_j\,B_j(\emptyset)A_j^{\frac{1}{\theta}}(\emptyset)\Big[B_j(\emptyset)A_j^{\frac{1}{\theta}}(\emptyset) - \frac{1}{\theta}\ln A_j(\emptyset) - \ln(1+$$

$$B_j(\emptyset))\Big] + r_j\big[A_{j-1}(\emptyset) - A_j(\emptyset)\big]^{-1}\Big\{jB_j(\emptyset)A_j^{1+\frac{1}{\theta}}(\emptyset)\Big[1 + (\theta+1)\Big[\ln(1+$$

$$B_j(\emptyset)) - B_j(\emptyset)A_j^{\frac{1}{\theta}}(\emptyset)\Big] + \frac{1}{\theta}\ln A_j(\emptyset)\Big] + \Big[j\,B_j(\emptyset)A_j^{1+\frac{1}{\theta}}(\emptyset) - (j-$$

$$1)B_{j-1}(\emptyset)A_{j-1}^{1+\frac{1}{\theta}}(\emptyset)\Big]\Big[\theta\left(A_j(\emptyset)\left(\ln(1+B_j(\emptyset)) + B_j(\emptyset)A_j^{\frac{1}{\theta}}(\emptyset)\right) - A_{j-1}(\emptyset)\left(\ln(1+$$

$$B_j(\emptyset)) + B_{j-1}(\emptyset)A_{j-1}^{\frac{1}{\theta}}(\emptyset)\right)\right) - 1\Big]\Big\}\Big\},$$

$$I_{22} = -\frac{\partial^2 \ln L(\Phi)}{\partial c_2^2} = \frac{\theta}{c_2^2}\sum_{j=1}^{k}\Big\{\left(\frac{c_2}{\theta}\right)^2\big(r_j + R_j\big)v_j^{2p}A_j^{\frac{-2}{\theta}}(\varepsilon) - R_j B_j^2(\emptyset)A_j^{\frac{2}{\theta}}(\emptyset) -$$

$$r_j\big[A_{j-1}(\emptyset) - A_j(\emptyset)\big]^{-1}\Big\{\Big[B_j(\emptyset)A_j^{1+\frac{1}{\theta}}(\emptyset)\Big[1 - (\theta+1)B_j(\emptyset)A_j^{\frac{1}{\theta}}(\emptyset)\Big] -$$

$$B_{j-1}(\emptyset)A_{j-1}^{1+\frac{1}{\theta}}(\emptyset)\Big[1 - (\theta+1)B_{j-1}(\emptyset)A_{j-1}^{\frac{1}{\theta}}(\emptyset)\Big]\Big] + \Big[\theta\big[A_{j-1}(\emptyset) -$$

$$A_j(\emptyset)\big]^{-1}\left(B_j(\emptyset)A_j^{1+\frac{1}{\theta}}(\emptyset) - B_{j-1}(\emptyset)A_{j-1}^{1+\frac{1}{\theta}}(\emptyset)\right) - 1\Big]\Big[B_j(\emptyset)A_j^{1+\frac{1}{\theta}}(\emptyset) -$$

$$B_{j-1}(\emptyset)A_{j-1}^{1+\frac{1}{\theta}}(\emptyset)\Big]\Big\}\Big\},$$

$$I_{23} = -\frac{\partial^2 \ln L(\Phi)}{\partial c_2 \partial p} = \sum_{j=1}^{k} lnv_j \left\{ (r_j + R_j)v_j^p A_j^{\frac{-1}{\theta}}(\varepsilon) \left[\frac{c_2 v_j^p}{\theta} A_j^{\frac{-1}{\theta}}(\varepsilon) - 1 \right] + \right.$$

$$\frac{\theta}{c_2} \left[R_j B_j(\emptyset)A_j(\omega)A_j^{\frac{1}{\theta}}(\emptyset) \left(1 - B_j(\emptyset)A_j^{\frac{1}{\theta}}(\emptyset) \right) - r_j[A_{j-1}(\emptyset)-A_j(\emptyset)]^{-1} \left\{ \left[B_j(\emptyset)A_j(\omega)A_j^{1+\frac{1}{\theta}}(\emptyset) \left[1 - \right. \right. \right. \right.$$

$$(\theta + 1)B_j(\emptyset)A_j^{\frac{1}{\theta}}(\emptyset) \right] - B_{j-1}(\emptyset)A_{j-1}^{1+\frac{1}{\theta}}(\emptyset)A_{j-1}(\omega) \left[1 - (\theta + 1)B_{j-1}(\emptyset)A_{j-1}^{\frac{1}{\theta}}(\emptyset) \right] \right] -$$

$$\theta[A_{j-1}(\emptyset) - A_j(\emptyset)]^{-1} \left(B_j(\emptyset)A_j^{1+\frac{1}{\theta}}(\emptyset) - B_{j-1}(\emptyset)A_{j-1}^{1+\frac{1}{\theta}}(\emptyset) \right) \left(B_j(\emptyset)A_j^{1+\frac{1}{\theta}}(\emptyset)A_j(\omega) - \right.$$

$$\left. \left. \left. B_{j-1}(\emptyset)A_{j-1}^{1+\frac{1}{\theta}}(\emptyset)A_{j-1}(\omega) \right) \right\} \right] \right\},$$

$$I_{24} = -\frac{\partial^2 \ln L(\Phi)}{\partial c_2 \partial \theta} = \sum_{j=1}^{k} \left\{ \frac{1}{\theta^2} (r_j + R_j)v_j^p A_j^{\frac{-1}{\theta}}(\varepsilon) \left[lnA_j(\varepsilon) + \theta ln \left(1 + \frac{c_2 v_j^p}{\theta} \right) - \frac{c_2 v_j^p}{A_j(\varepsilon)} \right] + \right.$$

$$\frac{R_j B_j(\emptyset)A_j^{\frac{1}{\theta}}(\emptyset)}{c_2} \left[B_j(\emptyset)A_j^{\frac{1}{\theta}}(\emptyset) - \frac{1}{\theta} \ln A_j(\emptyset) - \ln(1 + B_j(\emptyset)) \right] -$$

$$\frac{\theta r_j}{c_2 \left(A_{j-1}(\emptyset)-A_j(\emptyset) \right)} \left\{ \frac{1}{\theta^2} \left[B_j(\emptyset)A_j^{1+\frac{1}{\theta}}(\emptyset) \left[\theta(\theta + 1) \left(B_j(\emptyset)A_j^{\frac{1}{\theta}}(\emptyset) - \ln(1 + B_j(\emptyset)) \right) - \right. \right. \right.$$

$$\ln A_j(\emptyset) - \theta \right] - B_{j-1}(\emptyset)A_{j-1}^{1+\frac{1}{\theta}}(\emptyset) \left[\theta(\theta + 1) \left(B_{j-1}(\emptyset)A_{j-1}^{\frac{1}{\theta}}(\emptyset) - \ln(1 + B_{j-1}(\emptyset)) \right) - \right.$$

$$\ln A_{j-1}(\emptyset) - \theta \right] \right] - \left(B_j(\emptyset)A_j^{1+\frac{1}{\theta}}(\emptyset) - B_{j-1}(\emptyset)A_{j-1}^{1+\frac{1}{\theta}}(\emptyset) \right) \left[\left(A_{j-1}(\emptyset) - \right. \right.$$

$$A_j(\emptyset) \right)^{-1} \left(A_j(\emptyset) \left(\ln \left(1 + B_j(\emptyset) \right) - B_j(\emptyset)A_j^{\frac{1}{\theta}}(\emptyset) \right) - A_{j-1}(\emptyset) \left(\ln \left(1 + B_j(\emptyset) \right) - \right. \right.$$

$$\left. \left. \left. \left. B_{j-1}(\emptyset)A_{j-1}^{\frac{1}{\theta}}(\emptyset) \right) \right) - \frac{1}{\theta} \right] \right\} \right\},$$

$$I_{33} = -\frac{\partial^2 \ln L(\Phi)}{\partial p^2} = \theta \sum_{j=1}^{k} lnv_j^2 \left\{ R_j B_j(\emptyset)A_j^{\frac{1}{\theta}}(\emptyset) \left[A_j^2(\omega) \left(1 - B_j(\emptyset)A_j^{\frac{1}{\theta}}(\emptyset) \right) + jc_1 v_j^p \right] - \right.$$

$$\frac{(r_j+R_j)c_2 v_j^p}{\theta A_j^{\frac{1}{\theta}}(\varepsilon)} \left(1 - \frac{c_2 v_j^p}{\theta A_j^{\frac{1}{\theta}}(\varepsilon)} \right) - r_j[A_{j-1}(\emptyset) - A_j(\emptyset)]^{-1} \left\{ \left[B_j(\emptyset)A_j^{1+\frac{1}{\theta}}(\emptyset) \left[A_j^2(\omega) \left[1 - \right. \right. \right. \right.$$

$$(\theta + 1)B_j(\emptyset)A_j^{\frac{1}{\theta}}(\emptyset) \right] + jc_1 v_j^p \right] - B_{j-1}(\emptyset)A_{j-1}^{1+\frac{1}{\theta}}(\emptyset) \left[A_{j-1}^2(\omega) \left[1 - (\theta + \right. \right.$$

$$1)B_{j-1}(\emptyset)A_{j-1}^{\frac{1}{\theta}}(\emptyset)\Big] + (j-1)c_1 v_j^p\Big]\Big] - \theta\big[A_{j-1}(\emptyset) - A_j(\emptyset)\big]^{-1}\Big[B_j(\emptyset)A_j(\omega)A_j^{1+\frac{1}{\theta}}(\emptyset) -$$

$$B_{j-1}(\emptyset)A_{j-1}(\omega)A_{j-1}^{1+\frac{1}{\theta}}(\emptyset)\Big]^2\Big\}\bigg\},$$

$$I_{34} = -\frac{\partial^2 \ln L(\varPhi)}{\partial p \partial \theta} = \sum_{j=1}^{k} \ln v_j \Bigg\{ R_j\, B_j(\emptyset)A_j^{\frac{1}{\theta}}(\emptyset)A_j(\omega)\Big[B_j(\emptyset)A_j^{\frac{1}{\theta}}(\emptyset) - \tfrac{1}{\theta}\ln A_j(\emptyset) - \ln(1+$$

$$B_j(\emptyset))\Big] - \frac{(r_j + R_j)c_2 v_j^p}{\theta^2 A_j^{\frac{1}{\theta}}(\varepsilon)}\Bigg[\ln A_j(\varepsilon) - \theta \ln\left(1 + \frac{c_2 v_j^p}{\theta}\right) + \frac{c_2 v_j^p}{A_j^{\frac{1}{\theta}}(\varepsilon)}\Bigg] - r_j\big[A_{j-1}(\emptyset) -$$

$$A_j(\emptyset)\big]^{-1}\bigg\{B_j(\emptyset)A_j(\omega)A_j^{1+\frac{1}{\theta}}(\emptyset)\Big[(\theta+1)\left(B_j(\emptyset)A_j^{\frac{1}{\theta}}(\emptyset) - \ln(1+B_j(\emptyset))\right) - \tfrac{1}{\theta}\ln A_j(\emptyset) -$$

$$1\Big] - B_{j-1}(\emptyset)A_{j-1}(\omega)A_{j-1}^{1+\frac{1}{\theta}}(\emptyset)\Big[(\theta+1)\left(B_{j-1}(\emptyset)A_{j-1}^{\frac{1}{\theta}}(\emptyset) - \ln(1+B_{j-1}(\emptyset))\right) -$$

$$\tfrac{1}{\theta}\ln A_{j-1}(\emptyset) - 1\Big] + \Big[B_j(\emptyset)A_j^{1+\frac{1}{\theta}}(\emptyset)A_j(\omega) - B_{j-1}(\emptyset)A_{j-1}^{1+\frac{1}{\theta}}(\emptyset)A_{j-1}(\omega)\Big]\Big[1 -$$

$$\theta\big[A_{j-1}(\emptyset) - A_j(\emptyset)\big]^{-1}\left(A_j(\emptyset)\left(\ln(1+B_j(\emptyset)) - B_j(\emptyset)A_j^{\frac{1}{\theta}}(\emptyset)\right) - A_{j-1}(\emptyset)\left(\ln(1+\right.\right.$$

$$\left.\left.B_{j-1}(\emptyset)) - B_{j-1}(\emptyset)A_{j-1}^{\frac{1}{\theta}}(\emptyset)\right)\right)\Big]\bigg\}\Bigg\},$$

$$I_{44} = -\frac{\partial^2 \ln L(\Phi)}{\partial \theta^2}$$

$$= \frac{1}{\theta} \sum_{j=1}^{k} \left\{ R_j\, B_j(\emptyset) \left[A_j^{\frac{1}{\theta}}(\emptyset) \left(1 - B_j(\emptyset) A_j^{\frac{1}{\theta}}(\emptyset) + \frac{1}{\theta} lnA_j(\emptyset) + \ln(1 \right. \right. \right.$$

$$\left. + B_j(\emptyset)) \right) - (1 + B_j(\emptyset))^{-1} \Big]$$

$$+ \frac{(r_j + R_j)c_2 v_j^p}{\theta^2} \left[A_j^{\frac{-1}{\theta}}(\varepsilon) \left(\frac{1}{\theta} lnA_j(\omega) - \ln\left(1 + \frac{c_2 v_j^p}{\theta} \right) + \frac{c_2 v_j^p}{\theta A_j^{\frac{1}{\theta}}(\varepsilon)} - 1 \right) \right.$$

$$\left. + \left(1 + \frac{c_2 v_j^p}{\theta} \right)^{-1} \right]$$

$$+ r_j\big[A_{j-1}(\emptyset) - A_j(\emptyset)\big]^{-1} \left\{ \left[A_j(\emptyset) \left(\ln\left(1 + B_j(\emptyset) \right) - B_j(\emptyset) A_j^{\frac{1}{\theta}}(\emptyset) \right)^2 \right. \right.$$

$$+ \frac{1}{\theta} B_j(\emptyset) A_j(\emptyset) \left[\left(1 + B_j(\emptyset) \right)^{-1} \right.$$

$$\left. - A_j^{\frac{1}{\theta}}(\emptyset) \left[1 - B_j(\emptyset) A_j^{\frac{1}{\theta}}(\emptyset) + \frac{1}{\theta} lnA_j(\emptyset) + \ln\left(1 + B_j(\emptyset) \right) \right] \right] \right]$$

$$- A_{j-1}(\emptyset) \left(\ln\left(1 + B_{j-1}(\emptyset) \right) - B_{j-1}(\emptyset) A_{j-1}^{\frac{1}{\theta}}(\emptyset) \right)^2$$

$$+ \frac{1}{\theta} B_{j-1}(\emptyset) A_{j-1}(\emptyset) \left[\left(1 + B_{j-1}(\emptyset) \right)^{-1} \right.$$

$$\left. - A_{j-1}^{\frac{1}{\theta}}(\emptyset) \left[1 - B_{j-1}(\emptyset) A_j^{\frac{1}{\theta}}(\emptyset) + \frac{1}{\theta} lnA_{j-1}(\emptyset) + \ln\left(1 + B_{j-1}(\emptyset) \right) \right] \right]$$

$$+ \big[A_{j-1}(\emptyset) - A_j(\emptyset) \big]^{-1} \left(A_j(\emptyset) \left(\ln(1 + B_j(\emptyset)) - B_j(\emptyset) A_j^{\frac{1}{\theta}}(\emptyset) \right) \right.$$

$$\left. - A_{j-1}(\emptyset) \left(\ln(1 + B_{j-1}(\emptyset)) - B_{j-1}(\emptyset) A_{j-1}^{\frac{1}{\theta}}(\emptyset) \right) \right)^2 \right\} \right\},$$

4. EM- Algorithm

EM-algorithm is broadly applicable approach to the iterative computation of maximum Likelihood estimates (MLE), it was described and analyzed by Dempster, Laird, and Rubin (1977). The EM methods can be explained most easily in terms of a random sample that consists of two components, one observed and one unobserved or missing. Missing data occurs in life-testing, when, for example, a number of electrical units are switched on and the time when each fails is recorded. In such an experiment, it is usually necessary to curtail the recordings prior to the failure of all units. The failure times of the

units still working are unobserved, but the number of censored observations and the time of the censoring obviously provide information about the distribution of the failure times. Many common applications of EM methods do involve missing-data problems, but this is not necessary. Often, an EM method can be constructed based on two steps- called the Expectation step or the E-step and the maximization step or the M-step. Therefore, the algorithm is called the EM algorithm. The basic idea of the EM-algorithm is associate with the given incomplete-data, a complete-data problem. For which Maximum Likelihood (ML) estimation is computationally more tractable. For instance, the complete-data problem for chosen may yield a closed form solution to the (MLE) or may be amenable to MLE computation with a standard computer package. The methodology of the EM algorithm consists of reformulating the problem in terms of this more easily solved complete-data problem, establishing a relationship between the likelihoods of these two problems and exploiting the simpler MLE computation of the complete-data problem in the M-step of the iteration computing algorithm. Under k-level constant stress, the pdf of the truncated GL distribution will be applied in Q-function of EM-algorithm. Hence, For the E-step, we calculate $\Phi(\underline{\emptyset}, \widehat{\underline{\emptyset}}_m)$, where $\underline{\emptyset} = (c_1, c_2, p, \theta)$, and $\widehat{\underline{\emptyset}}_m = (\hat{c}_{1m}, \hat{c}_{2m}, \hat{p}_m, \hat{\theta}_m)$ in m^{th} iteration. Thus, the Q-function of the truncated GL distribution can be obtained as follows,

$$\Phi(\underline{\emptyset}, \widehat{\underline{\emptyset}}_m) = E(lnL, \widehat{\underline{\emptyset}}_m) = \int lnL \, f(t)dt,$$

therefore, the Q-function is

$$\Phi(\underline{\emptyset}, \widehat{\underline{\emptyset}}_m) = \sum_{j=1}^{k} \{R_j E(lnA_j(\emptyset), \widehat{\underline{\emptyset}}_m) + (r_j + R_j)lnA_j(\varepsilon)$$
$$+ r_j E(ln[A_{j-1}(\emptyset) - A_j(\emptyset)], \widehat{\underline{\emptyset}}_m)\}. \tag{10}$$

For the M-Step, we note that the maximization can be done by finding the solution of

$$E\left(\frac{\partial lnL}{\partial \emptyset}, \widehat{\underline{\emptyset}}_m\right) = \frac{\partial \Phi(\underline{\emptyset}, \widehat{\underline{\emptyset}}_m)}{\partial \emptyset} = 0 \tag{11}$$

Then, we obtain the estimators of (c_1, c_2, p, θ) by differentiating Eq.(10) with respect to (c_1, c_2, p, θ) respectively and equating to zero. The E and M steps are repeatedly until the estimates of parameters converges. Therefore, we use the MAThCad program to solve the nonlinear equations simultaneously to obtain $(\hat{c}_1, \hat{c}_2, \hat{p}, \hat{\theta})$ as will be seen in Section 5.

5. Numerical Results

A numerical study is conducted to assess the performance of the proposed algorithm in Section (3) and Section (4). The a progressively Type-I censored grouped sample under constant stress test is considered to obtain the unknown parameters $(\hat{c}_1, \hat{c}_2, \hat{p}, \hat{\theta})$ by solving the system of nonlinear equations get from Eqs. (6-9). Then the values which getten of estimators are used to get the absolute relative bias (ARBias), and mean square error (MSE). Furthermore, Fisher information matrix, the scale parameters and the reliability function. Tables (1) to (3) summarize the results of the simulation study based on different values of N, R and $v_1 = 1, v_2 = 2$.

From the results, the following observations can be made on the performance of CSALT parameter estimation of the two scales truncated GL lifetime distribution by EM-algorithm. We observe the maximum likelihood estimators of the parameters $c_{10} = 0.4$, $c_{20} = 0.001$, $p_0 = 1.990$, $\theta_0 = 7.70$, have good statistical properties than the second set of parameters $c_{10} = 0.4$, $c_{20} = 0.001$, $p_0 = 1.993$, $\theta_0 = 7.51$ for the same sample size (see Table 1). Also, as the sample size increases the RABiases and MSEs of the estimated parameters decrease. This indicates that the maximum likelihood estimates provide asymptotically normally distributed and consistent estimators for the parameters. In addition, the asymptotic variances of the estimators are decreasing when the sample size increasing (see Table2). Moreover, the reliability function decreases when the mission time x_0 increases, and there is an inverse proportional relationship between $\hat{\alpha}_{1u}$, $\hat{\alpha}_{2u}$ and $\hat{R}_u(x_0)$ at the same mission time (see Table3).

Table 1:   The MLE, ARBias, and MSE of the Parameters

$c_{10} = 0.4$, $c_{20} = 0.001$, $p_0 = 1.990$, $\theta_0 = 7.70$						$c_{10} = 0.4$, $c_{20} = 0.001$, $p_0 = 2.0$, $\theta_0 = 7.51$				
para.	N	R	*MLE*	*ARBias*	*MSE*	N	R	*MLE*	*ARBias*	*MSE*
c_1	30	20	0.4005	0.0013	2.9E-7	66	42	0.4000	0.0009	1.3E-7
c_2			0.0008	0.1559	2.4E-8			0.0009	0.0970	9.4E-9
p			1.9793	0.0054	1.2E-4			1.9880	0.0027	3.0E-5
θ			7.6509	0.0064	6.4E-3			7.5320	0.0029	4.7E-4
$c_{10} = 0.4$, $c_{20} = 0.001$, $p_0 = 1.993$, $\theta_0 = 7.51$						$c_{10} = 0.4$, $c_{20} = 0.001$, $p_0 = 1.993$, $\theta_0 = 7.51$				
MSE	N	R	*MLE*	*ARBias*	*MSE*	N	R	*MLE*	*ARBias*	*MSE*
c1	30	20	0.40090	0.0022	8.0E-7	97	68	0.40000	3.7E-5	2.1E-10
c2			0.00083	0.1700	2.9E-8			0.00099	1.3	1.8E-12
p			1.98190	0.0056	1.2E-4			2.00000	0.000063	1.6E-8
			7.59390	0.0112	7.0E-3			7.51100	0.000100	6.2E-7
$c_{10} = 0.4$, $c_{20} = 0.001$, $p_0 = 2.0$, $\theta_0 = 6.54$						$c_{10} = 0.4$, $c_{20} = 0.001$, $p_0 = 1.992$, $\theta_0 = 7.52$				
MSE	N	R	*MLE*	*ARBias*	*MSE*	N	R	*MLE*	*ARBias*	*MSE*
c1	47	30	0.40520	0.0130000	2.7E-5	186	118	0.40080	0.0021	6.9E-7
c2			0.00099	0.0780000	6.1E-9			0.00087	0.1313	1.7E-8
p			2.00000	0.0001929	1.5E-7			1.99200	0.0036	5.1E-5
			6.51800	0.0034090	5.0E-4			7.46850	0.0069	2.7E-3

Table 2: Estimated Variance-Covariance Matrix

$c_{10}=0.4$, $c_{20}=0.001$, $p_0=1.990$, $\theta_0=7.70$						$c_{10}=0.4$, $c_{20}=0.001$, $p_0=2.0$, $\theta_0=7.51$					
N	R	$\hat{c}_1$	$\hat{c}_2$	$\hat{p}$	$\hat{\theta}$	N	R	$\hat{c}_1$	$\hat{c}_2$	$\hat{p}$	$\hat{\theta}$
30	20	0.1441	-0.9892	0.0184	0.00034	66	42	0.1971	-0.7374	0.4070	0.0061
			0.0791	-0.1261	-0.0237				0.0739	-0.1517	-0.2770
				2.3E-3	4.4E-5					0.0840	0.0013
					8.2E-7						1.9E-5
$c_{10}=0.4$, $c_{20}=0.001$, $p_0=1.993$, $\theta_0=7.51$						$c_{10}=0.4$, $c_{20}=0.001$, $p_0=1.993$, $\theta_0=7.51$					
N	R	$\hat{c}_1$	$\hat{c}_2$	$\hat{p}$	$\hat{\theta}$	N	R	$\hat{c}_1$	$\hat{c}_2$	$\hat{p}$	$\hat{\theta}$
30	20	0.138	-0.9881	0.0170	0.4E-4	97	68	0.1192	-0.1286	0.526	0.0093
			0.0776	-0.1218	-0.0248				0.0940	-0.161	-0.0029
				0.0021	-4.3E-5					0.066	0.0011
					8.7E-7						2.1E-5
$c_{10}=0.4$, $c_{20}=0.001$, $p_0=2.0$, $\theta_0=6.54$						$c_{10}=0.4$, $c_{20}=0.001$, $p_0=1.992$, $\theta_0=7.52$					
N	R	$\hat{c}_1$	$\hat{c}_2$	$\hat{p}$	$\hat{\theta}$	N	R	$\hat{c}_1$	$\hat{c}_2$	$\hat{p}$	$\hat{\theta}$
47	30	0.1224	-0.2969	0.147	0.0034	186	118	0.1292	-0.3192	0.8824	0.0175
			0.0200	-0.357	0.0083				0.0397	-0.3862	-0.7677
				0.0180	0.0004					0.1068	0.0021
					0.1E-6						4.2E-5

Table 3: Estimated $\hat{\alpha}_{1u}$, $\hat{\alpha}_{2u}$ and $\hat{R}_u(x_0)$

$c_{10}=0.4$, $c_{20}=0.001$, $p_0=1.990$, $\theta_0=7.70$						$c_{10}=0.4$, $c_{20}=0.001$, $p_0=2.0$, $\theta_0=7.51$					
N	R	$\hat{\alpha}_{1u}$	$\hat{\alpha}_{2u}$	x_0	$\hat{R}_u(x_0)$	N	R	$\hat{\alpha}_{1u}$	$\hat{\alpha}_{2u}$	x_0	$\hat{R}_u(x_0)$
30	20	0.2267	0.00048	10	0.9949	66	42	0.226	0.00051	10	0.9946
				25	0.8572					25	0.8501
				30	0.6249					30	0.6108
				40	0.0240					40	0.0213
$c_{10}=0.4$, $c_{20}=0.001$, $p_0=1.993$, $\theta_0=7.51$						$c_{10}=0.4$, $c_{20}=0.001$, $p_0=1.993$, $\theta_0=7.51$					
N	R	$\hat{\alpha}_{1u}$	$\hat{\alpha}_{2u}$	x_0	$\hat{R}_u(x_0)$	N	R	$\hat{\alpha}_{1u}$	$\hat{\alpha}_{2u}$	x_0	$\hat{R}_u(x_0)$
30	20	0.2267	0.00047	10	0.9950	97	68	0.225	0.00056	10	0.9940
				25	0.8593					25	0.8400
				30	0.6295					30	0.5900
				40	0.0253					40	0.0180
$c_{10}=0.4$, $c_{20}=0.001$, $p_0=2.0$, $\theta_0=6.54$						$c_{10}=0.4$, $c_{20}=0.001$, $p_0=1.992$, $\theta_0=7.52$					
N	R	$\hat{\alpha}_{1u}$	$\hat{\alpha}_{2u}$	x_0	$\hat{R}_u(x_0)$	N	R	$\hat{\alpha}_{1u}$	$\hat{\alpha}_{2u}$	x_0	$\hat{R}_u(x_0)$
47	30	0.2280	0.00052	10	0.9943	186	118	0.226	0.00049	10	0.9948
				25	0.8384					25	0.8556
				30	0.5839					30	0.6225
				40	0.0166					40	0.0242

6. Conclusion

Truncated distributions arise when sample selection is not possible in some sub-region of the sample space. For example, the GLD is considered inappropriate for modeling lifetime data because left hand side of its distribution extends to negative infinity, and this could conceivably result in modeling negative times-to-failure. This has necessitated the use of truncated GLD truncated at point zero for modeling lifetime data. So, this paper is assumed that the stress is effected of two scale parameters of the truncated generalized logistic distribution, and dealt with constant-stress accelerated life testing in the case of progressive Type-I censored grouped data. Maximum likelihood estimators of the model parameters and the associated Fisher Information matrix are derived by used the EM algorithm method. Also, the estimator of the two scale parameters, and the reliability function under usual conditions are derived.

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