Exponential Distribution as a Stress-Strength Model with Type-I Censored Data from a Bayesian Viewpoint

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Abstract

In this paper, the Bayes estimate is derived for the parameters of the exponential model. The estimate is obtained using the squared error loss and LINEX loss function. The risk with the estimate of α, β under LINEX loss function has been made. Finally, numerical study is given to illustrate the results.

Keywords: Exponential distribution, Bayes' estimator, LINEX loss function, Reliability function, Stress-strength model, Risk function, Squared error loss function.

Acronyms and Abbreviations

CDF	Cumulative distribution function
PDF	Probability density function
MLE	Maximum likelihood estimator
UMVUE	Uniformly minimum variance unbiased estimator
MSE	Mean square error

Notations

$F_1(:)$	Cumulative distribution function of X
$F_2(:)$	Cumulative distribution function of Y
Δ	Convex loss function
$h(S)$	Density function of S
$E[L(\Delta)]$	Expected value of $L(\Delta)$
$R_L(\hat{\alpha}, \hat{\beta})$	Joint risk efficiency of $\hat{\alpha}$ and $\hat{\beta}$

1. Introduction

Let X be the strength of the component which is subjected to stress Y where X and Y are random variables distributed as exponential distribution with parameter α and β, respectively. The probability density functions of X and Y is given by:

$$f(x;\alpha)=\frac{1}{\alpha}e^{-\frac{x}{\alpha}},\ x>0,\qquad \alpha>0 \tag{1}$$

$$f(y;\beta)=\frac{1}{\beta}\mathrm{e}^{-\frac{y}{\beta}},\ y>0,\qquad \beta>0 \tag{2}$$

And their cumulative distribution function

$$F(x;\alpha)=(1-e^{-\frac{x}{\alpha}}). \tag{3}$$

$$F(y;\beta)=(1-\mathrm{e}^{-\frac{y}{\beta}}). \tag{4}$$

The exponential distribution has very wide application in life testing problems. Where, many authors have been studied the stress-strength model with exponential distribution. For example, Tong (1977) had a look at the estimation of $\mathrm{P(Y>X)}$ for exponential families. Beg (1980 a, b and c) estimated the exponential family, two parameter exponential distribution and truncation parameter distributions. Sathe and Shah (1981) studied estimation of $\mathrm{Pr(Y>X)}$ for the exponential distribution. Awad and Charraf (1986) studied three different estimators for reliability that have a bivariate exponential distribution. Kunchur and Mounoli (1993) obtained UMVUE of stress- strength model for multi component survival model based on exponential distribution for parallel system. Shrinkage estimation of $\mathrm{Pr(Y\le X)}$ in the exponential case has been discussed by Ayman and Walid (2003). Canfield (2009) found bayesian estimate of reliability for the exponential case is developed which utilizes the basic notion of loss in estimation theory.

In the estimation of reliability function, use of symmetric loss function may be inappropriate as has been recognized by Canfield (1970) and Varian (1975). Zellner (1986) proposed an asymmetric loss function known as the LINEX loss function, which has been found to be appropriate in the situation where overestimation is more serious under-estimation or vice-versa. the LINEX loss function that has both linear and exponential components and is appropriate to represent asymmetric losses. This loss function grows approximately exponentially on one side of zero, the value of the estimation error, and approximately linearly on the other side. The LINEX loss function assigns unequal weights to the underestimation and overestimation by using a shape parameter. For small values of the shape parameter, the LINEX loss function is approximately symmetric and not too far from the quadratic loss function. The LINEX loss function is more general than the squared error loss function as the later is a special case of the former. Also, in recent time there are a lot of authors studied LINEX loss functions for different distribution. Such as, Jaheen (2004) studied the exponential model based on record statistics, where the Bayes estimator using the square loss function and LINEX loss function. Bayes estimator of exponential distribution studied again in (2006) by Cuirong (et al) to find the risk function. Also, Sanku (2007) derived Bayes estimators for inverted exponential distribution and obtained squared error and LINEX loss functions. Based on upper record values, the Bayes estimators for the parameter of Rayleigh distribution are obtained. These estimators are obtained.

On the basis of square error and LINEX (linear-exponential) loss functions was studied by Hendi, Abu-Youssef and Alraddadi (2007).

Suppose $\Delta_1 = \frac{\hat{\alpha}}{\alpha} - 1$, where $\hat{\alpha}$ an estimate of is α. Consider the following convex loss function.

$$L(\Delta_1) = e^{c_1\Delta_1} - c_1\Delta_1 - 1; c_1 \neq 0 \tag{5}$$

The sign and magnitude of 'c_1' represent, respectively, the direction and degree of asymmetry. A positive value of 'c_1'is used when overestimation is more costly than underestimation; while a negative value of 'c_1' is used in the reverse situation. For 'c_1' close to zero, this loss function is approximately squared error loss and therefore almost symmetric. Several authors (Basu & Ebrahimi, 1991; Rojo, 1987; Soliman, 2000; Zellner, 1986) have used this loss function in various estimation and prediction problems.

Also, the same definition for β where $\Delta_2 = \frac{\hat{\beta}}{\beta} - 1$ and $\hat{\beta}$ is an estimate β. Consider the following convex loss function.

$$L(\Delta_2) = e^{c_2\Delta_2} - c_2\Delta_2 - 1; c_2 \neq 0 \tag{6}$$

The sign of 'c_2' is treated like 'c_1'.

2. Bayes' Estimate of α and β

The estimation of the unknown parameters α and β are concerned where α and β are exponential distribution based on a type-I censored random sample of size n and m, respectively. The likelihood function of α is given by:

$$L(x_1,..., x_n \mid \alpha) = (\frac{1}{\alpha})^U e^{\frac{-\sum_{i=1}^{n} x_i\lambda_i - T_1(n-U)}{\alpha}} \tag{7}$$

And for β

$$L(y_1,..., y_m \mid \beta) = (\frac{1}{\beta})^V e^{\frac{-\sum_{j=1}^{m} y_j\gamma_j - T_2(m-V)}{\beta}} \tag{8}$$

Where U is random. These lifetimes observed only when $x_i \le T_1,\ i=1,...,n$. Therefore

$$\lambda_i = \begin{cases} 1 & \text{if} \quad x_i \le T_1 \\ 0 & \text{if} \quad x_i > T_1 \end{cases}$$

Where $U = \sum_{i=1}^{n} \lambda_i$, and for V is random. Where these lifetimes observed only when $y_j \le T_2,\ j = 1,...,\ m$. Therefore

$$\gamma_j = \begin{cases} 1 & \text{if} \quad y_j \le T_2 \\ 0 & \text{if} \quad y_j > T_2 \end{cases}$$

Where $V = \sum_{j=1}^{m} \gamma_j$.

Put $S_1 = \sum_{i=1}^{n} x_i \lambda_i + T_1(n-U)$ and $S_2 = \sum_{j=1}^{m} y_j \gamma_j + T_2(m-V)$

Because x_i and y_j ; $i=1,...,n$ and $j=1,...,m$ are independent and identically distributed exponential random variables with parameters α and β, respectively. It is also known that a sum of independent exponentially distributed random variables gives a Gamma distributed variable where the probability density functions of S_1 and S_2 is

$$h(S_1) = \frac{1}{\Gamma(n+1)} \left(\frac{1}{\alpha \lambda_i}\right)^{n+1} S_1^{\ n} e^{-\frac{S_1}{\lambda_i \alpha}},\ S_1 > 0 \tag{9}$$

and

$$h(S_2) = \frac{1}{\Gamma(m+1)} \left(\frac{1}{\beta \gamma_j}\right)^{m+1} S_2^{\ m} e^{-\frac{S_2}{\gamma_j \beta}},\ S_2 > 0 \tag{10}$$

It follows, from (7) and (9), that the posterior density of α, for a given x, is given by

$$\pi_1 = \frac{e^{\frac{-(\sum_{i=1}^{n} x_i \lambda_i + b_1 + T_1(n-U))}{\alpha}} (b_1 + \sum_{i=1}^{n} x_i \lambda_i + T_1(n-U))^{a_1+U}}{\alpha^{a_1+U-1} \Gamma(a_1+U)},\ a_1, b_1 > 0 \tag{11}$$

Under the square error loss function, the Bayes estimator of α, denoted by $\hat{\alpha}_{MSB\ 1}$, is the mean of posterior distribution that can be shown to be

$$\hat{\alpha}_{MSB\ 1} = \frac{b_1 + S_1}{a_1 + U - 1} \tag{12}$$

Also, from (8) and (10), the posterior density of β, for a given y, is given by

$$\pi_2 = \frac{e^{\frac{-(\sum_{j=1}^{m} y_j\gamma_j + b_2 + T_2(m-V))}{\alpha}} (b_2 + \sum_{j=1}^{m} y_j\gamma_j + T_2(m-V))^{a_2+V}}{\beta^{a_2+V-1}\Gamma(a_2+V)}, a_2, b_2 > 0 \quad (13)$$

Under the square error loss function, the Bayes estimator of β, denoted by $\hat{\beta}_{MSB\ 2}$, is the mean of posterior distribution that can be shown to be

$$\hat{\beta}_{MSB\ 2} = \frac{b_2 + S_2}{a_2 + V - 1} \quad (14)$$

2.1 Bayes' Estimator of α and β, based on LINEX Loss Function

Under the LINEX loss function (5), the posterior expectation of the loss function $L(\Delta_1)$ with respect to $\pi_1(\alpha | x)$ in (11) for the case of α

$$\begin{aligned} E[L(\Delta_1)] &= \int_0^\infty (e^{c_1(\frac{\hat{\alpha}}{\alpha}-1)} - c_1(\frac{\hat{\alpha}}{\alpha}-1)-1)\pi_1(\alpha \mid x)\mathrm{d}\alpha \\ &= e^{-c_1}E(e^{c_1(\frac{\hat{\alpha}}{\alpha})}) - c_1E(\frac{\hat{\alpha}}{\alpha}) - 1 \end{aligned} \quad (15)$$

The value of $\hat{\alpha}$ that minimizes the posterior expectation of the loss function $L(\Delta_1)$ denoted by $\hat{\alpha}_{LB1}$ is obtained by solving equation

$$\frac{\partial E[L(\Delta_1)]}{\partial \hat{\alpha}} = e^{-c_1}E(\frac{c_1}{\alpha}e^{c_1(\frac{\hat{\alpha}}{\alpha})}) - c_1E(\frac{1}{\alpha}) = 0 \quad (16)$$

That is, $\hat{\alpha}_{LB1}$ is the solution of the equation

$$E(\frac{1}{\alpha}e^{c_1(\frac{\hat{\alpha}}{\alpha})}) = e^{c_1}E(\frac{1}{\alpha}) \quad (17)$$

The optimal estimate of $\hat{\alpha}$ relative to $L(\Delta_1)$ is found by using (11) and (17)

$$\hat{\alpha}_{LB1} = \frac{(b_1+S_1)}{c_1}(1-e^{\frac{-c_1}{a_1+U+1}}) \quad (18)$$

Similarly, the LINEX loss function (6), the posterior expectation of the loss function $L(\Delta_2)$ with respect to $\pi_2(\beta | y)$ in (12) for the case of β, the value of $\hat{\beta}$ that minimizes the posterior expectation of the loss function $L(\Delta_2)$ denoted by $\hat{\beta}_{LB2}$ is

$$\hat{\beta}_{LB2} = \frac{(b_2+S_2)}{c_2}(1-e^{\frac{-c_2}{a_2+V+1}}) \quad (19)$$

2.2 The joint risk efficiency of $\hat{\alpha}_{LB1}$ and $\hat{\beta}_{LB2}$ with respect to $\hat{\alpha}_{MSB\ 1}$ and $\hat{\beta}_{MSB\ 2}$ under LINEX Loss $L(\Delta_1)$ and $L(\Delta_2)$

The risk functions of estimators $\hat{\alpha}_{LB1}$ and $\hat{\beta}_{LB2}$ relative to $L(\Delta_1)$ and $L(\Delta_2)$ is of interest. These joint risk functions are denoted by $R_L(\hat{\alpha}_{LB1},\hat{\beta}_{LB2})$ and $R_L(\hat{\alpha}_{MSB1},\hat{\beta}_{MSB2})$, where subscript L denotes risk relative to $L(\Delta_1)$ and $L(\Delta_2)$ and are given by using $h(S_1)$ and $h(S_2)$.

The joint risk efficiency of $\hat{\alpha}_{LB1}$ and $\hat{\beta}_{LB2}$ are studied by found, we will find $R_L(\hat{\alpha}_{LB1},\hat{\beta}_{LB2})$ as follow

$$
\begin{aligned}
R_L(\hat{\alpha}_{LB1},\hat{\beta}_{LB2}) &= E_{XY}\,(L(\Delta_1), L(\Delta_2)) \\
&= \int_0^\infty\int_0^\infty L(\Delta_1)L(\Delta_2)h(S_1)h(S_2)\,dS_1\,dS_2 \\
&= \int_0^\infty [e^{c_1(\frac{\hat{\alpha}_{LB1}}{\alpha}-1)} - c_1(\frac{\hat{\alpha}_{LB1}}{\alpha}-1)-1]h(S_1)\,dS_1 \\
&\quad \int_0^\infty [e^{c_2(\frac{\hat{\beta}_{LB2}}{\beta}-1)} - c_2(\frac{\hat{\beta}_{LB2}}{\beta}-1)-1]h(S_2)\,dS_2
\end{aligned}
\tag{20}
$$

$$
\begin{aligned}
R_L(\hat{\alpha}_{LB1},\hat{\beta}_{LB2}) = {} & [\frac{e^{\frac{b_1(1-e^{\frac{-c_1}{a_1+U+1}})}{\alpha}-c_1}}{(1-(1-e^{\frac{-c_1}{a_1+U+1}})\lambda_i)^{n+1}} - (1-e^{\frac{-c_1}{a_1+U+1}})(\frac{b_1}{\alpha}+(n+1))+c_1-1] \\
& [\frac{e^{\frac{b_2(1-e^{\frac{-c_2}{a_2+V+1}})}{\beta}-c_2}}{(1-(1-e^{\frac{-c_2}{a_2+V+1}})\gamma_j)^{m+1}} - (1-e^{\frac{-c_2}{a_2+V+1}})(\frac{b_2}{\beta}+(m+1))+c_2-1]
\end{aligned}
\tag{21}
$$

In the same manner

$$R_L(\hat{\alpha}_{MSB\ 1},\hat{\beta}_{MSB\ 2}) = E_{XY}\ (L(\Delta_1), L(\Delta_2))$$
$$= \int_0^\infty\int_0^\infty L(\Delta_1)L(\Delta_2)h(S_1)h(S_2)dS_1 dS_2$$
$$= \int_0^\infty [e^{c_1(\frac{\hat{\alpha}_{MSB\ 1}}{\alpha}-1)} - c_1(\frac{\hat{\alpha}_{MSB\ 1}}{\hat{\alpha}}-1)-1]h(S_1)dS_1 \quad (22)$$
$$\int_0^\infty [e^{c_2(\frac{\hat{\beta}_{MSB\ 2}}{\beta}-1)} - c_2(\frac{\hat{\beta}_{MSB\ 2}}{\beta}-1)-1]h(S_2)dS_2$$

$$R_L(\hat{\alpha}_{MSE\ 1},\hat{\beta}_{MSE\ 2}) = [\frac{(a_1+U-1)^{n+1}\lambda_i e^{c_1(\frac{b_1}{\alpha(a_1+U-1)}-1)}}{(c_1\lambda_i-(a_1+U-1))^{n+1}}$$
$$-\frac{c_1}{(a_1+U-1)}(b_1\alpha^{-1}+(n+1))+c_1-1]$$
$$[\frac{(a_2+V-1)^{m+1} e^{c_2(\frac{b_2}{\beta(a_2+V-1)}-1)}}{(c_2\gamma_j-(a_2+V-1))^{m+1}} \quad (23)$$
$$-\frac{c_2}{(a_2+V-1)}(b_2\beta^{-1}+(m+1))+c_2-1]$$

The risk efficiency of $R_L(\hat{\alpha}_{LB1},\hat{\beta}_{LB2})$ with respect to $R_L(\hat{\alpha}_{MSB1},\hat{\beta}_{MSB2})$ under LINEX Loss $L(\Delta_1)$ and $L(\Delta_2)$ may be defined as follows:

$$RE1_L(\hat{\alpha}_{LB1},\hat{\beta}_{LB2},\hat{\alpha}_{MSB1},\hat{\beta}_{MSB2}) = \frac{R_L(\hat{\alpha}_{MSB1},\hat{\beta}_{MSB2})}{R_L(\hat{\alpha}_{LB1},\hat{\beta}_{LB2})} \quad (24)$$

3. Numerical Examples

To compare the proposed estimator $\hat{\alpha}_{LB1}$ and $\hat{\beta}_{LB2}$ with the estimator $\hat{\alpha}_{MSB\ 1}$ and $\hat{\beta}_{MSB\ 2}$, the risk functions are computed so as to see whether $R_L(\hat{\alpha}_{LB1},\hat{\beta}_{LB2})$ out performs $R_L(\hat{\alpha}_{MSB1},\hat{\beta}_{MSB2})$ under LINEX loss $L(\Delta_1)$ and $L(\Delta_2)$. A comparison of this type may be needed to check whether an estimator is inadmissible under some loss functions. If so, the estimator would not be used for the losses specified by that loss function. For this purpose the risks of the estimators and risk efficiency have been computed $RE1_L$. N=1000 samples are generated of sizes $n,m = 5,10,15 \text{ and } 20$ and $\alpha = 2$ and

$\beta = 2,3$. The results are presented in tables (1-8) where $c_1 = c_2 = 0.8, 1$ and -1 and our time is T=30 hours and 50 hours.

It is evident from that Expect for Table (1) at $c_1 = 0.8,\ c_2 = 0.8$, $\alpha = 2$, $\beta = 3$ has another values because when $n = m$ the values of $RE\,1_L$ is increasing as n, m increasing. Also, in all the risk efficiency $RE\,1_L$ is greater than one for the sample sizes $n, m = 5,10,15 \text{ and } 20$.

4. Conclusion

Our observations about the results are stated in the following points:

1. The values of $RE\,1_L$ is decreasing when n, m increase. But, for $n \neq m$ the values of $RE\,1_L$ is decreasing as n, m increasing.
2. for tables 2,3,6 and 8 the value of the risk efficiency $RE\,1_L$ is greater than one for the sample sizes $n, m = 5,10,15 \text{ and } 20$ especially when $n = m$, for all values of c_1, c_2, which indicates that the proposed estimators $R_L(\hat{\alpha}_{LB1}, \hat{\beta}_{LB2})$ is preferable to $R_L(\hat{\alpha}_{MSB1}, \hat{\beta}_{MSB2})$.
3. for tables 2,4,5 and 7 the value of the risk efficiency $RE\,1_L$ is lower than one for the sample sizes $n, m = 5,10,15 \text{ and } 20$ especially when $n \neq m$, for all values of c_1, c_2, which indicates that the proposed estimators $R_L(\hat{\alpha}_{MSB1}, \hat{\beta}_{MSB2})$ is preferable to $R_L(\hat{\alpha}_{LB1}, \hat{\beta}_{LB2})$.

T=30 hours

Table (1)

The estimators $\hat{\alpha}_{LB1}$, $\hat{\beta}_{LB2}$, $\hat{\alpha}_{MSB\ 1}$ and $\hat{\beta}_{MSB\ 2}$ with respect to $L(\Delta_1)$ and $L(\Delta_2)$, the risk efficiencies $R_L(\hat{\alpha}_{LB1}, \hat{\beta}_{LB2})$ and $R_L(\hat{\alpha}_{MSB1}, \hat{\beta}_{MSB2})$ for the values of $c_1 = 0.8, c_2 = 0.8$, $\alpha = 2$, $\beta = 3$ and $R = 0.4$

n	m	$\hat{\alpha}_{MSB\ 1}$	$\hat{\beta}_{MSB\ 2}$	$\hat{\alpha}_{LB1}$	$\hat{\beta}_{LB2}$	R_{LB}	R_{MSB}	$RE1$
5	5	1.784	2.975	1.438	2.399	2.47E-04	1.51E-04	0.613
10	10	2.498	2.93	2.153	2.526	4.12E-04	2.32E-04	0.562
15	15	2.651	2.767	2.367	2.471	8.03E-04	3.98E-04	0.496
20	20	2.587	3.433	2.36	3.132	2.05E-03	8.38E-04	0.409
5	4	2.079	2.88	1.676	2.56	2.321E-03	9.374E-04	0.404
10	8	2.095	2.426	1.805	2.547	9.552E-04	4.621E-04	0.484
10	9	2.16	2.397	1.862	2.34	8.729E-04	4.279E-04	0.49
20	18	2.134	2.766	1.947	2.504	2.730E-04	1.647E-04	0.603
20	19	2.1	3.092	1.99	2.811	2.594E-04	1.577E-04	0.608

T=30 hours
Table (2)

The estimators $\hat{\alpha}_{LB1}, \hat{\beta}_{LB2}$, $\hat{\alpha}_{MSB\ 1}$ and $\hat{\beta}_{MSB\ 2}$ with respect to $L(\Delta_1)$ and $L(\Delta_2)$, the risk efficiencies $R_L(\hat{\alpha}_{LB\ 1}, \hat{\beta}_{LB\ 2})$ and $R_L(\hat{\alpha}_{MSB1}, \hat{\beta}_{MSB2})$ for the values of $c_1 = 0.8, c_2 = 0.8$, $\alpha = 1$, $\beta = 2$ and $R = 0.333$

n	m	$\hat{\alpha}_{MSB\ 1}$	$\hat{\beta}_{MSB\ 2}$	$\hat{\alpha}_{LB1}$	$\hat{\beta}_{LB2}$	R_{LB}	R_{MSB}	$RE1$
5	5	1.136	1.597	0.916	1.45	5.29E-04	5.40E-04	1.017
10	10	1.33	2.371	1.146	2.044	3.37E-04	3.41E-04	1.011
15	15	1.283	1.0899	1.145	1.698	2.067E-04	2.076E-04	1.004
20	20	1.12	2.32	0.998	2.119	1.389E-04	1.391E-04	1.001
5	4	0.688	1.566	0.555	1.78	7.058E-04	5.905E-04	0.978
10	8	1.314	1.864	1.132	2.042	3.945E-04	3.831E-04	0.971
10	9	1.22	1.805	1.052	1.541	3.637E-04	3.606E-04	0.992
20	18	1.1	2.115	1.21	1.915	1.522E-04	1.499E-04	0.985
20	19	1.325	1.95	1.013	2.285	1.453E-04	1.443E-04	0.993

T=30 hours
Table (3)

The estimators $\hat{\alpha}_{LB1}, \hat{\beta}_{LB2}$, $\hat{\alpha}_{MSB\ 1}$ and $\hat{\beta}_{MSB\ 2}$ with respect to $L(\Delta_1)$ and $L(\Delta_2)$, the risk efficiencies $R_L(\hat{\alpha}_{LB\ 1}, \hat{\beta}_{LB\ 2})$ and $R_L(\hat{\alpha}_{MSB1}, \hat{\beta}_{MSB2})$ for the values of $c_1 = 1, c_2 = 1$, $\alpha = 1$, $\beta = 2$ and $R = 0.333$

n	m	$\hat{\alpha}_{MSB\ 1}$	$\hat{\beta}_{MSB\ 2}$	$\hat{\alpha}_{LB1}$	$\hat{\beta}_{LB2}$	R_{LB}	R_{MSB}	$RE1$
5	5	1.273	2.47	1.167	1.975	4.05E-04	5.60E-04	1.681
10	10	1.254	2.192	1.037	1.813	5.35E-04	9.58E-04	1.51
15	15	1.626	2.247	1.417	1.958	1.13E-03	2.00E-03	1.564
20	20	1.3	2.233	1.18	2.005	1.54E-03	1.60E-03	1.045
5	4	0.982	2.077	0.785	2.328	1.811E-03	3.397E-03	1.005
10	8	1.474	2.104	1.136	2.001	9.589E-04	9.527E-04	0.993
10	9	1.609	1.836	1.379	1.557	8.842E-04	8.974E-04	1.015
20	18	1.325	2.363	1.244	2.131	3.701E-04	3.705E-04	1.001
20	19	1.415	2.571	1.324	2.328	3.533E-04	3.567E-04	1.01

T=30 hours
Table (4)

The estimators $\hat{\alpha}_{LB1}, \hat{\beta}_{LB2}$, $\hat{\alpha}_{MSB\ 1}$ and $\hat{\beta}_{MSB\ 2}$ with respect to $L(\Delta_1)$ and $L(\Delta_2)$, the risk efficiencies $R_L(\hat{\alpha}_{LB\ 1}, \hat{\beta}_{LB\ 2})$ and $R_L(\hat{\alpha}_{MSB1}, \hat{\beta}_{MSB2})$ for the values of $c_1 = -1, c_2 = -1$, $\alpha = 1, \beta = 2$ and $R = 0.333$

n	m	$\hat{\alpha}_{MSB\ 1}$	$\hat{\beta}_{MSB\ 2}$	$\hat{\alpha}_{LB1}$	$\hat{\beta}_{LB2}$	R_{LB}	R_{MSB}	$RE1$
5	5	1.156	1.764	1.004	1.533	3.50E-04	3.22E-04	0.919
10	10	1.356	1.957	1.232	1.779	5.23E-04	4.75E-04	0.909
15	15	1.256	2.212	1.02	2.057	8.51E-04	7.63E-04	0.896
20	20	1.114	2.6	1.12	2.453	1.55E-03	1.38E-03	0.893
5	4	1.343	1.652	1.167	1.417	1.744E-03	1.507E-03	0.819
10	8	1.12	2.254	1.245	2.02	1.002E-03	8.691E-04	0.867
10	9	1.364	2.304	1.232	2.094	9.205E-04	8.126E-04	0.883
20	18	1.17	2.554	1.017	2.603	3.845E-04	3.489E-04	0.908
20	19	1.253	2.664	1.24	2.507	3.667E-04	3.350E-04	0.914

T=50 hours
Table (5)

The estimators $\hat{\alpha}_{LB1}, \hat{\beta}_{LB2}$, $\hat{\alpha}_{MSB\ 1}$ and $\hat{\beta}_{MSB\ 2}$ with respect to $L(\Delta_1)$ and $L(\Delta_2)$, the risk efficiencies $R_L(\hat{\alpha}_{LB\ 1}, \hat{\beta}_{LB\ 2})$ and $R_L(\hat{\alpha}_{MSB1}, \hat{\beta}_{MSB2})$ for the values of $c_1 = 0.8, c_2 = 0.8$, $\alpha = 2, \beta = 3$ and R=0.4

n	m	$\hat{\alpha}_{MSB\ 1}$	$\hat{\beta}_{MSB\ 2}$	$\hat{\alpha}_{LB1}$	$\hat{\beta}_{LB2}$	R_{LB}	R_{MSB}	$RE1$
5	5	1.502	2.49	1.835	2.3	5.98E-04	3.65E-04	0.61
10	10	2.025	2.95	1.735	2.16	9.97E-04	5.57E-04	0.559
15	15	2.351	3.396	2.09	3.018	1.93E-03	9.53E-04	0.493
20	20	2.354	2.751	2.139	2.501	5.07E-03	2.95E-03	0.486
5	4	1.823	3.345	1.457	3.158	5.54E-03	2.22E-03	0.4
10	8	2.234	2.99	1.914	3.011	2.30E-03	1.10E-03	0.48
10	9	2.701	3.0247	1.955	3.089	2.10E-03	1.02E-03	0.487
20	18	2.123	2.798	2.455	2.533	5.61E-04	3.97E-04	0.6
20	19	2.373	3.1	2.165	2.818	2.59E-04	1.58E-04	0.608

T=50 hours
Table (6)

The estimators $\hat{\alpha}_{LB1}, \hat{\beta}_{LB2}$, $\hat{\alpha}_{MSB\ 1}$ **and** $\hat{\beta}_{MSB\ 2}$ **with respect to** $L(\Delta_1)$ **and** $L(\Delta_2)$**, the risk efficiencies** $R_L(\hat{\alpha}_{LB\ 1}, \hat{\beta}_{LB\ 2})$ **and** $R_L(\hat{\alpha}_{MSB1}, \hat{\beta}_{MSB2})$ **for the values of** $c_1 = 0.8, c_2 = 0.8$ **,** $\alpha = 2$ **,** $\beta = 2$ **and R=0.5**

n	m	$\hat{\alpha}_{MSB\ 1}$	$\hat{\beta}_{MSB\ 2}$	$\hat{\alpha}_{LB1}$	$\hat{\beta}_{LB2}$	R_{LB}	R_{MSB}	$RE1$
5	5	0.991	1.83	0.799	1.475	5.29E-04	5.40E-04	1.017
10	10	1.471	2.325	1.268	2.405	3.37E-04	3.41E-04	1.011
15	15	0.132	1.923	1.222	1.717	2.067E-04	2.076E-04	1.004
20	20	0.999	2.36	1.011	2.153	1.389E-04	1.391E-04	1.001
5	4	0.923	1.635	0.744	1.717	7.058E-04	5.905E-04	0.978
10	8	1.775	2.669	1.53	2.253	3.945E-04	3.831E-04	0.971
10	9	1.7	1.929	1.245	1.647	3.637E-04	3.606E-04	0.992
20	18	1.333	2.454	1.111	2.222	1.522E-04	1.499E-04	0.985
20	19	1.532	2.478	1.212	2.323	1.453E-04	1.443E-04	0.993

T=50 hours
Table (7)

The estimators $\hat{\alpha}_{LB1}, \hat{\beta}_{LB2}$, $\hat{\alpha}_{MSB\ 1}$ **and** $\hat{\beta}_{MSB\ 2}$ **with respect to** $L(\Delta_1)$ **and** $L(\Delta_2)$**, the risk efficiencies** $R_L(\hat{\alpha}_{LB\ 1}, \hat{\beta}_{LB\ 2})$ **and** $R_L(\hat{\alpha}_{MSB1}, \hat{\beta}_{MSB2})$ **for the values of** $c_1 = 1, c_2 = 1$ **,** $\alpha = 1$ **,** $\beta = 2$ **and** $R = 0.333$

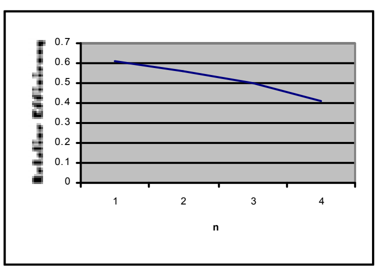
n	m	$\hat{\alpha}_{MSB\ 1}$	$\hat{\beta}_{MSB\ 2}$	$\hat{\alpha}_{LB1}$	$\hat{\beta}_{LB2}$	R_{LB}	R_{MSB}	$RE1$
5	5	0.725	1.784	0.579	1.524	1.54E-03	1.60E-03	1.045
10	10	0.888	1.967	0.657	1.685	8.20E-04	8.48E-04	1.034
15	15	0.965	2.452	0.875	2.179	5.03E-04	5.15E-04	1.024
20	20	1.121	2.314	0.924	2.014	3.38E-04	3.44E-04	1.017
5	4	1.149	2.131	0.899	1.704	1.720E-03	1.729E-03	1.005
10	8	1.1545	2.553	0.983	2.141	9.589E-04	9.527E-04	0.993
10	9	1.254	2.178	0.999	1.866	8.842E-04	8.974E-04	1.015
20	18	1.222	2.236	1.021	2.017	3.701E-04	3.705E-04	1.088
20	19	1.235	2.353	1.0222	2.131	3.533E-04	3.567E-04	1.01

T=50 hours
Table (8)

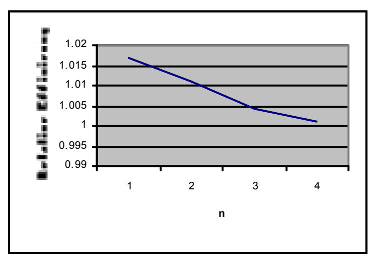
The estimators $\hat{\alpha}_{LB1}, \hat{\beta}_{LB2}$, $\hat{\alpha}_{MSB\ 1}$ and $\hat{\beta}_{MSB\ 2}$ with respect to $L(\Delta_1)$ and $L(\Delta_2)$, the risk efficiencies $R_L(\hat{\alpha}_{LB\ 1}, \hat{\beta}_{LB\ 2})$ and $R_L(\hat{\alpha}_{MSB1}, \hat{\beta}_{MSB2})$ for the values of $c_1 = -1, c_2 = -1$, $\alpha = 1$, $\beta = 2$ and $R = 0.333$

n	m	$\hat{\alpha}_{MSB\ 1}$	$\hat{\beta}_{MSB\ 2}$	$\hat{\alpha}_{LB1}$	$\hat{\beta}_{LB2}$	R_{LB}	R_{MSB}	$RE1$
5	5	1.533	1.841	1.332	1.6	3.50E-04	3.22E-04	0.919
10	10	1.232	2.031	1.222	1.846	5.23E-04	4.75E-04	0.909
15	15	1.325	2.194	1.25	2.041	8.51E-04	7.63E-04	0.896
20	20	1.424	2.015	1.24477	2.145	1.55E-03	1.38E-03	0.893
5	4	1.254	2.159	1.33	1.876	1.744E-03	1.507E-03	0.864
10	8	1.111	2.162	1.145	1.937	1.002E-03	8.691E-04	0.867
10	9	1.675	1.831	1.522	1.654	9.205E-04	8.126E-04	0.883
20	18	1.254	2.428	1.425	2.279	3.845E-04	3.489E-04	0.908
20	19	1.92	2.921	1.811	2.749	3.667E-04	3.350E-04	0.914

$T = 30$



$c_1 = 0.8, c_2 = 0.8$
$\alpha = 2$, $\beta = 3$



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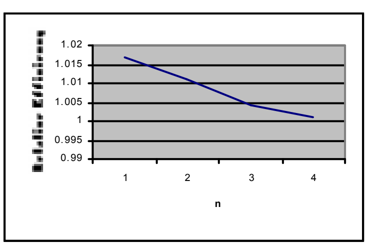
$\alpha = 1$, $\beta = 2$



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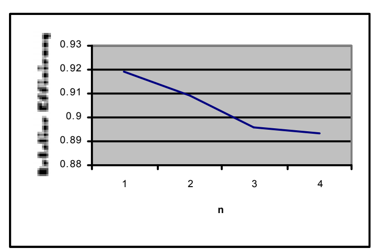

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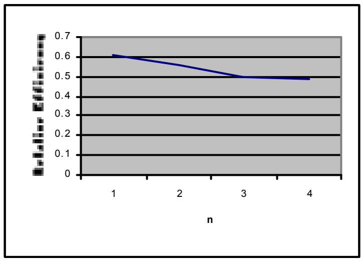
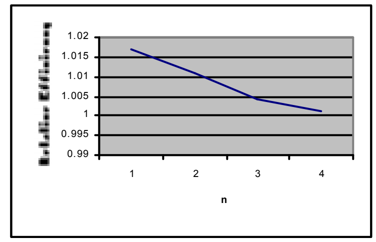
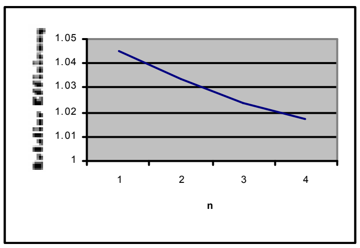
$$T = 50$$



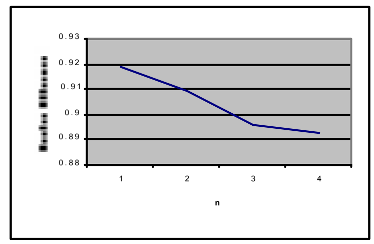
$c_1 = 0.8, c_2 = 0.8$
$\alpha = 2, \beta = 3$



$c_1 = 0.8, c_2 = 0.8$
$\alpha = 1, \beta = 2$



$c_1 = 1, c_2 = 1$
$\alpha = 1, \beta = 2$



$c_1 = -1, c_2 = -1$
$\alpha = 1, \beta = 2$

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7. References

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