Kurtosis Correction Method for Variable Control Charts – a Comparison in Laplace Distribution

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Abstract

A variable quality characteristic is assumed to follow the well known Laplace Distribution. Control chart constants for the process mean, process dispersion based on a number of sub group statistics including sub group mean and range are evaluated from the first principles. Limits obtained through kurtosis correction method are borrowed from Tadikamalla and Popescu (2003). The performance of these sets of control limits is compared through a simulation study and the relative preferences are arrived at. The methods are illustrated by an example.

1. Introduction

The probability density function of a standard Laplace distribution is given by

$$G(x) = \frac{1}{2}\exp\{-|X|\} \quad -\infty < x < \infty \tag{1.1}$$

It is well known that the coefficient of kurtosis of Laplace distribution is 6. Generally the variable control chart constants are available when the process variate follows normal distribution. In some situations like positive valued skewed distributions also control chart constants are available. For example, Edgeman (1989) – Inverse Gaussian Distribution, Kantam and Sriram (2001) – Gamma and Exponential Distribution, Kantam et al (2006) – Log-Logistic Distribution, and references there in. In this paper, a symmetric non normal model – the double exponential as specified in the equation (1.1) is considered to be the distribution of a variable quality characteristic. Tadikamalla and Popescu (2003) suggested kurtosis correction method for $\overline{X}$ and R charts in symmetric distributions. The percentiles of subgroup statistics like sample mean and range are worked out by Subba Rao and Kantam (2008) to develop the control chart constants for $\overline{X}$ and R charts. The present paper makes a comparative study of the control chart constants developed by Tadikamalla and Popescu (2003), Subba Rao and Kantam (2008). The rest of the paper is organized as follows: Section 2 discusses briefly the summary of Tadikamalla and Popescu (2003), Subba Rao and Kantam (2008). Section 3 presents the comparative study.

2. Control Chart Constants

i) $\overline{X}$ and R chart from Kurtosis Correction Method:

Let X_{ij}, i=1,2,…,k j=1,2,…,n be an observation on a variable quality characteristic divided into k subgroups of size n each. If $\overline{X}_i$, R_i, $\overline{\overline{X}}$, $\overline{R}$ respectively denote the i^{th} subgroup mean, range, grand mean, mean of ranges. Based on the sampling distribution of Karl Pearson's sample coefficient of Kurtosis, Tadikamalla and Popescu (2003) developed control chart constants for $\overline{X}$ chart and R chart as given below:

$$\overline{X}-\text{Chart}: \begin{aligned} UCL_{\overline{X}} &= \overline{\overline{X}} + \left(3 + \frac{k_4(\overline{X})}{1+0.33.k_4(\overline{X})}\right)\frac{\overline{R}}{d_2^*\sqrt{n}} = \overline{\overline{X}} + A_2^*\overline{R} \\ CL_{\overline{X}} &= \overline{\overline{X}} \\ LCL_{\overline{X}} &= \overline{\overline{X}} - \left(3 + \frac{k_4(\overline{X})}{1+0.33.k_4(\overline{X})}\right)\frac{\overline{R}}{d_2^*\sqrt{n}} = \overline{\overline{X}} - A_2^*\overline{R} \end{aligned} \quad (2.1)$$

$$R-\text{Chart}: \begin{aligned} UCL_R &= \overline{R} + \left(3 + \frac{d_4^*}{1+0.33d_4^*}\right)\frac{\overline{R}d_3^*}{d_2^*} \\ &= \overline{R}\left(1 + \left(3 + \frac{d_4^*}{1+0.33d_4^*}\right)\frac{d_3^*}{d_2^*}\right) = \overline{R}D_4^* \\ CL_R &= \overline{R} \\ LCL_R &= \overline{R} - \left(3 + \frac{d_4^*}{1+0.33d_4^*}\right)\frac{\overline{R}d_3^*}{d_2^*} \\ &= \overline{R}\left(1 - \left(3 + \frac{d_4^*}{1+0.33d_4^*}\right)\frac{d_3^*}{d_2^*}\right) = \overline{R}D_3^* \end{aligned} \quad (2.2)$$

They have tabulated A_2^*, D_3^*, D_4^* which depend on the coefficient of kurtosis of the model of the study. Their tabulated values can be used for any symmetric model with a coefficient of kurtosis different from 3. Accordingly they named the constants as kurtosis corrected control chart constants.

ii) $\overline{X}$ Chart from First Principle:

Let L_1 and L_2 be the two equitailed 99.73% percentiles in a Laplace distribution. Because of symmetry, it can be seen that

$$P(-5.9145<Z<5.9145) = 0.9973 \quad (2.3)$$

where Z is a standard Laplace variate. If μ, σ are location and scale parameters then μ±5.9145σ gives a 99.73% probability interval for the double exponential variate. If $\overline{X}$ is the sample mean of n observations from double exponential distribution

$$E(\overline{X}) \pm 5.9145 \text{ S.E } (\overline{X}) \tag{2.4}$$

be used to find the confidence limits for the sample mean. Though the distribution of $\overline{X}$ is not symmetric they have suggested the limits as parallel to those suggested by Shewart for mean and range charts based on order statistics for normal population in order to make a comparative study. They have also estimated the process mean μ by sample mean. The S.E of sample mean is given by

$$\text{S.E}(\overline{X}) = \frac{\sigma}{\sqrt{n}} \sqrt{\sum_{i=1}^{n} \sum_{j=1}^{n} \text{COV}(Z_i, Z_j)} \tag{2.5}$$

where Z_i, Z_j are respectively i^{th}, j^{th} standard order statistics in a sample of size n from double exponential distribution and $COV(Z_i,Z_j)$ are available in Govindarajullu (1966). An estimate of the standard error is obtained by estimating σ of (2.5) with the help of $\overline{R}/(\alpha_{(n)} - \alpha_{(1)})$, where $\overline{R}$ is mean of sample ranges over repeated sampling, $\alpha_{(n)}$, $\alpha_{(1)}$ are the expected values of n^{th} and 1^{st} order statistics in a standard double exponential distribution. From (2.3) and (2.4) over repeated sampling of size n, we can write down the control limits of $\overline{X}$-chart for process mean as

$$\overline{\overline{X}} + A_2^* \overline{R} \tag{2.6}$$

where A_2^* is an expression that depends on moments of order statistics in sample of size n from standard double exponential distribution and hence depends on n. They have tabulated these values for various values of n = 2 (1) 10.

iii) R-chart from First Principles:

The process dispersion is estimated by sample range in order to construct the control limits for range chart from the pair of limits

$$E(R) \pm 5.9145 \text{ S.E } (R) \tag{2.7}$$

The expectation and standard error in (2.7) can be computed using the moments of order statistics given in Govindarajullu (1966).

The L.C.L is taken as zero when ever it turnout to be negative. Accordingly the L.C.L and U.C.L are denoted as $D_3^* \overline{R}$, $D_4^* \overline{R}$ over repeated sampling of same size n. They have tabulated the values of D_3^* and D_4^*.

3. Comparison

The sets of control limits for the subgroup mean and subgroup range in the two approaches namely kurtosis correction method and first principles can be compared with the help of the coverage probabilities of the control limits. This is done through simulation of samples of size 1000 of subgroup size 2(1)10. For each subgroup statistics $\overline{X}_i$, R_i, $\overline{\overline{X}}$, $\overline{R}$ are calculated to construct the control limits using the constants of the two approaches. The proportion of $\overline{X}_i$'s, R_i that have fallen in the two pairs of control limits for the respective subgroup statistics are arrived at .These are given in the following table.

	$\overline{X}$ - Chart		R - Chart	
n	Subba Rao & Kantam (2008) Limits	Kurtosis Corrected Limits	Subba Rao & Kantam (2008) Limits	Kurtosis Corrected Limits
2	1.00	0.9992	1.00	0.9992
3	1.00	0.9994	1.00	0.9981
4	1.00	1.00	0.9992	0.9992
5	1.00	1.00	0.999	1.00
6	1.00	0.9975	1.00	1.00
7	1.00	1.00	1.00	1.00
8	1.00	1.00	1.00	1.00
9	1.00	0.9982	1.00	1.00
10	1.00	0.998	1.00	1.00

This table shows that though the coverage probabilities in both the cases are quite significant the limits obtained from the first principles have an edge over those obtained from kurtosis correction method. This paper thus exemplifies that "a correction is a correction, exactness is superior though marginal".

References

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