Improved Ratio Estimators for the Population Mean Using Non-Conventional Measures of Dispersion

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Abstract

In recent times, it is common to the make use of auxiliary information to increase the precision of estimators in sample surveys. In this study, we propose some new modified linear regression type ratio estimators for estimating population mean by some non-conventional dispersion measures such as: Gini's mean difference, Downton's method and probability weighted moments with linear combination of population correlation coefficient and population coefficient of variation. Expressions for the bias and the mean squared error are derived and are compared with those of the usual ratio estimator and the existing ratio type estimators in literature. Conditions are determined for which the proposed estimators perform better than the existing estimators. Both theoretical and empirical findings show the soundness of the proposed procedure for estimation of population mean.

Keywords: Auxiliary variable, Downton's technique, Gini's mean difference, Probability weighted moments.

1. Introduction

In survey research, there are situations when the information, on every unit in the population, is available. If a variable, that is known for every unit of the population and is not a variable of direct interest but instead employed to improve the sampling plan or to enhance the estimation of the variables of interest, is called an auxiliary variable. The auxiliary information is commonly associated with the use of ratio type estimation methods and to improve the efficiency of the estimators in survey sampling.

Consider a finite population $U = \{U_1, U_2, U_3, \ldots, U_N\}$ of N distinct and identifiable units. Let Y be the study variable with value Y_i measured of U_i , $i = 1,2, \ldots, N$ giving a vector $Y = \{Y_1, Y_2, Y_3, \dots, Y_N\}$. The objective is to estimate population mean $\bar{Y} = \frac{1}{N}\sum_{i=1}^{N} Y_i$ on the basis of a random sample. When the population parameters of the auxiliary variable, such as population mean, kurtosis, skewness, coefficient of variation, median, quartiles, correlation coefficient, deciles etc., are known, ratio estimators and their modifications are available in the literature which perform better than the usual sample mean under the simple random sampling without replacement (SRSWOR).

The notations used in this paper can be described as follows:

NOMENCLATURE

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N	Population size
n	Sample size
$f = {}^{n}/_{N}$	Sampling fraction
Y	Study variable
X	Auxiliary variable
$\bar{X}, \bar{Y}$	Population means
$\bar{x}, \bar{y}$	Sample means
x, y	Sample totals
S_x, S_y	Population standard deviations
S_{xy}	Population covariance between X and Y
C_x, C_y	Coefficient of variation
$B(.)$	Bias of the Estimator
$MSE(.)$	Mean square error of the estimator
$\hat{\bar{Y}}_i$	Existing modified ratio estimator of $\bar{Y}$
$\hat{\bar{Y}}_{pj}$	Proposed modified ratio estimator of $\bar{Y}$
M_d	Median of X
$QD = \frac{Q_3 - Q_1}{2}$	Quartile Deviation
$G = \frac{4}{N-1}\sum_{i=1}^{N}\left(\frac{2i-N-1}{2N}\right) X_{(i)}$	Gini's Mean Difference
$D = \frac{2\sqrt{\pi}}{N(N-1)}\sum_{i=1}^{N}\left(i - \frac{N+1}{2}\right) X_{(i)}$	Downton's method
$S_{pw} = \frac{\sqrt{\pi}}{N^2}\sum_{i=1}^{N}(2i - N - 1)\, X_{(i)}$	Probability Weighted Moments

Subscript

i	For existing estimators
j	For proposed estimators

Greek

ρ	Coefficient of correlation
$\beta_1 = \frac{N\sum_{i=1}^{N}(X_i-\bar{X})^3}{(N-1)(N-2)S^3}$	Coefficient of skewness of auxiliary variable
$\beta_2 = \frac{N(N+1)\sum_{i=1}^{N}(X_i-\bar{X})^4}{(N-1)(N-2)(N-3)S^4} - \frac{3(N-1)^2}{(N-2)(N-3)}$	Coefficient of kurtosis of auxiliary variable
$b = \frac{S_{xy}}{S_x^2}$	Regression coefficient of Y on X

Based on the above mentioned notations, the mean ratio estimator for estimating the population mean, $\bar{Y}$, of the study variable Y is defined as

$$\hat{\bar{Y}}_r = \frac{\bar{y}}{\bar{x}}\bar{X} \tag{1}$$

The bias, related constant and the mean squared error (MSE) of the ratio estimator are respectively given by

$$B\left(\hat{\bar{Y}}_r\right) = \frac{(1-f)}{n}\frac{1}{\bar{X}}\left(RS_x^2 - \rho S_x S_y\right) \quad R = \frac{\bar{Y}}{\bar{X}} \quad MSE\left(\hat{\bar{Y}}_r\right) = \frac{(1-f)}{n}\left(S_y^2 + R^2 S_x^2 - 2R\rho S_x S_y\right)$$

The ratio estimator given in (1) is used for improving the precision of the estimate of the population mean as compared to usual sample mean estimator whenever a positive correlation exists between the study variable and the auxiliary variable. Cochran (1940) suggested a classical ratio type estimator for the estimation of finite population mean using one auxiliary variable under simple random sampling scheme. Murthy (1967) proposed a product type estimator to estimate the population mean or total of study variable by using auxiliary information when coefficient of correlation is negative. Rao (1991) introduced difference type ratio estimator that outperforms conventional ratio and linear regression estimators. Upadhyaya & Singh (1999) modified ratio type estimators using coefficient of variation and coefficient of kurtosis of the auxiliary variate. Singh & Tailor (2003) proposed a family of estimators using known values of some parameters by using SRSWOR for estimation of population mean of the study variable. Sisodiaa & Dwivedi (1981) and Singh et al. (2004) utilized coefficient of variation of the auxiliary variate. Further improvements are achieved by introducing a large number of modified ratio estimators with the use of known coefficient of variation, kurtosis, skewness, median, coefficient of correlation, decile (cf. Subramani and Kumarpandiyan, 2012 a,b,c and d).

The organization of the rest of the article is as follows: Section 2 provides a description of the existing estimators. The structure of suggested modified linear regression type ratio estimator and the efficiency comparison of the suggested estimator with the usual ratio estimator and the existing estimators are presented in Section 3. Section 4 consists of an empirical study of proposed estimators. Finally, Section 5 summarizes the findings of the study.

2. Existing Ratio Estimators

Kadilar and Cingi (2004) suggested ratio type estimators for the population mean in the simple random sampling using some known auxiliary information on coefficient of kurtosis and coefficient of variation. They showed that their suggested estimators are more efficient than traditional ratio estimator in the estimation of the population mean.

Kadilar & Cingi (2004) estimators are given by

$$\hat{\bar{Y}}_1 = \frac{\bar{y}+b(\bar{X}-\bar{x})}{\bar{x}}\bar{X},$$

$$\hat{\bar{Y}}_2 = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}+C_x)}(\bar{X}+C_x),$$

$$\hat{\bar{Y}}_3 = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}+\beta_2)}(\bar{X}+\beta_2),$$

$$\hat{\bar{Y}}_4 = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}\beta_2+C_x)}(\bar{X}\beta_2+C_x),$$

$$\hat{\bar{Y}}_5 = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}C_x+\beta_2)}(\bar{X}C_x+\beta_2).$$

The biases, related constants and the MSE for Kadilar and Cingi (2004) estimators are respectively as follows:

$$B\left(\hat{\bar{Y}}_1\right) = \frac{(1-f)}{n}\frac{S_x^2}{\bar{Y}}R_1^2, \quad R_1 = \frac{\bar{Y}}{\bar{X}} \qquad MSE\left(\hat{\bar{Y}}_1\right) = \frac{(1-f)}{n}\left(R_1^2S_x^2 + S_y^2(1-\rho^2)\right),$$

$$B\left(\hat{\bar{Y}}_2\right) = \frac{(1-f)}{n}\frac{S_x^2}{\bar{Y}}R_2^2, \quad R_2 = \frac{\bar{Y}}{(\bar{X}+C_x)} \qquad MSE\left(\hat{\bar{Y}}_2\right) = \frac{(1-f)}{n}\left(R_2^2S_x^2 + S_y^2(1-\rho^2)\right),$$

$$B\left(\hat{\bar{Y}}_3\right) = \frac{(1-f)}{n}\frac{S_x^2}{\bar{Y}}R_3^2, \quad R_3 = \frac{\bar{Y}}{(\bar{X}+\beta_2)} \qquad MSE\left(\hat{\bar{Y}}_3\right) = \frac{(1-f)}{n}\left(R_3^2S_x^2 + S_y^2(1-\rho^2)\right),$$

$$B\left(\hat{\bar{Y}}_4\right) = \frac{(1-f)}{n}\frac{S_x^2}{\bar{Y}}R_4^2, \quad R_4 = \frac{\bar{Y}\beta_2}{(\bar{X}\beta_2+C_x)} \qquad MSE\left(\hat{\bar{Y}}_4\right) = \frac{(1-f)}{n}\left(R_4^2S_x^2 + S_y^2(1-\rho^2)\right),$$

$$B\left(\hat{\bar{Y}}_5\right) = \frac{(1-f)}{n}\frac{S_x^2}{\bar{Y}}R_5^2, \quad R_5 = \frac{\bar{Y}C_x}{(\bar{X}C_x+\beta_2)} \qquad MSE\left(\hat{\bar{Y}}_5\right) = \frac{(1-f)}{n}\left(R_5^2S_x^2 + S_y^2(1-\rho^2)\right).$$

Kadilar and Cingi (2006) developed some modified ratio estimators using known value of coefficient of correlation, kurtosis and coefficient of variation as follows:

$$\hat{\bar{Y}}_6 = \frac{\bar{y}+b(\bar{X}-\bar{x})}{\bar{x}}(\bar{X}+\rho),$$

$$\hat{\bar{Y}}_7 = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}C_x+\rho)}(\bar{X}C_x+\rho),$$

$$\hat{\bar{Y}}_8 = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}\rho+C_x)}(\bar{X}\rho+C_x),$$

$$\hat{\bar{Y}}_9 = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}\beta_2+\rho)}(\bar{X}\beta_2+\rho),$$

$$\hat{\bar{Y}}_{10} = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}\rho+\beta_2)}(\bar{X}\rho+\beta_2).$$

The biases, related constants and the MSE for Kadilar and Cingi (2006) estimators are respectively given by

$$B\left(\hat{\hat{Y}}_6\right)=\frac{(1-f)}{n}\frac{S_x^2}{\bar{Y}}R_6^2,\qquad R_6=\frac{\bar{Y}}{\bar{X}+\rho}\qquad MSE\left(\hat{\hat{Y}}_6\right)=\frac{(1-f)}{n}\left(R_6^2S_x^2+S_y^2(1-\rho^2)\right),$$

$$B\left(\hat{\hat{Y}}_7\right)=\frac{(1-f)}{n}\frac{S_x^2}{\bar{Y}}R_7^2,\qquad R_7=\frac{\bar{Y}C_x}{(\bar{X}C_x+\rho)}\qquad MSE\left(\hat{\hat{Y}}_7\right)=\frac{(1-f)}{n}\left(R_7^2S_x^2+S_y^2(1-\rho^2)\right),$$

$$B\left(\hat{\hat{Y}}_8\right)=\frac{(1-f)}{n}\frac{S_x^2}{\bar{Y}}R_8^2,\qquad R_8=\frac{\bar{Y}\rho}{(\bar{X}\rho+C_x)}\qquad MSE\left(\hat{\hat{Y}}_8\right)=\frac{(1-f)}{n}\left(R_8^2S_x^2+S_y^2(1-\rho^2)\right),$$

$$B\left(\hat{\hat{Y}}_9\right)=\frac{(1-f)}{n}\frac{S_x^2}{\bar{Y}}R_9^2,\qquad R_9=\frac{\bar{Y}\beta_2}{(\bar{X}\beta_2+\rho)}\qquad MSE\left(\hat{\hat{Y}}_9\right)=\frac{(1-f)}{n}\left(R_9^2S_x^2+S_y^2(1-\rho^2)\right),$$

$$B\left(\hat{\hat{Y}}_{10}\right)=\frac{(1-f)}{n}\frac{S_x^2}{\bar{Y}}R_{10}^2,\qquad R_{10}=\frac{\bar{Y}\rho}{(\bar{X}\rho+\beta_2)}\qquad MSE\left(\hat{\hat{Y}}_{10}\right)=\frac{(1-f)}{n}\left(R_{10}^2S_x^2+S_y^2(1-\rho^2)\right).$$

Yan and Tian (2010) proposed some modified ratio estimators using coefficient of skewness and kurtosis as follows:

$$\hat{\hat{Y}}_{11}=\frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}+\beta_1)}(\bar{X}+\beta_1),$$

$$\hat{\hat{Y}}_{12}=\frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}\beta_1+\beta_2)}(\bar{X}\beta_1+\beta_2).$$

The biases, related constants and the MSE for Yan and Tian (2010) estimators are respectively given by

$$B\left(\hat{\hat{Y}}_{11}\right)=\frac{(1-f)}{n}\frac{S_x^2}{\bar{Y}}R_{11}^2,\qquad R_{11}=\frac{\bar{Y}}{(\bar{X}+\beta_1)}\qquad MSE\left(\hat{\hat{Y}}_{11}\right)=\frac{(1-f)}{n}\left(R_{11}^2S_x^2+S_y^2(1-\rho^2)\right),$$

$$B\left(\hat{\hat{Y}}_{12}\right)=\frac{(1-f)}{n}\frac{S_x^2}{\bar{Y}}R_{12}^2,\qquad R_{12}=\frac{\bar{Y}\beta_1}{(\bar{X}\beta_1+\beta_2)}\qquad MSE\left(\hat{\hat{Y}}_{12}\right)=\frac{(1-f)}{n}\left(R_{12}^2S_x^2+S_y^2(1-\rho^2)\right).$$

Yan and Tian (2010) showed that the use of coefficient of skewness and coefficient of kurtosis, respectively, provides better estimates for the population mean in comparison to the usual ratio estimator and numerous existing estimators.

Subramani and Kumarapandiyan (2012a, 2012b, 2012c) introduced the following estimators with the use of population median, skewness, kurtosis and coefficient of variation of auxiliary information in the simple random sampling for the estimation of the population mean.

$$\hat{\hat{Y}}_{13}=\frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}+M_d)}(\bar{X}+M_d),$$

$$\hat{\hat{Y}}_{14}=\frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}C_x+M_d)}(\bar{X}C_x+M_d),$$

$$\hat{\hat{Y}}_{15}=\frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}\beta_1+M_d)}(\bar{X}\beta_1+M_d),$$

$$\hat{\hat{Y}}_{16}=\frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}\beta_2+M_d)}(\bar{X}\beta_2+M_d).$$

The biases, related constant and the MSE for Subramani and Kumarapandiyan (2012a, 2012b, 2012c) estimators are respectively given by

$$B\left(\hat{\hat{Y}}_{13}\right) = \frac{(1-f)}{n}\frac{S_x^2}{\bar{Y}}R_{13}^2, \qquad R_{13} = \frac{\bar{Y}}{(\bar{X}+M_d)} \qquad MSE\left(\hat{\hat{Y}}_{13}\right) = \frac{(1-f)}{n}\left(R_{13}^2 S_x^2 + S_y^2(1-\rho^2)\right),$$

$$B\left(\hat{\hat{Y}}_{14}\right) = \frac{(1-f)}{n}\frac{S_x^2}{\bar{Y}}R_{14}^2, \qquad R_{14} = \frac{\bar{Y}C_x}{(\bar{X}C_x+M_d)} \quad MSE\left(\hat{\hat{Y}}_{14}\right) = \frac{(1-f)}{n}\left(R_{14}^2 S_x^2 + S_y^2(1-\rho^2)\right),$$

$$B\left(\hat{\hat{Y}}_{15}\right) = \frac{(1-f)}{n}\frac{S_x^2}{\bar{Y}}R_{15}^2, \qquad R_{15} = \frac{\bar{Y}\beta_1}{(\bar{X}\beta_1+M_d)} \quad MSE\left(\hat{\hat{Y}}_{15}\right) = \frac{(1-f)}{n}\left(R_{15}^2 S_x^2 + S_y^2(1-\rho^2)\right),$$

$$B\left(\hat{\hat{Y}}_{14}\right) = \frac{(1-f)}{n}\frac{S_x^2}{\bar{Y}}R_{14}^2, \qquad R_{16} = \frac{\bar{Y}\beta_2}{(\bar{X}\beta_2+M_d)} \quad MSE\left(\hat{\hat{Y}}_{16}\right) = \frac{(1-f)}{n}\left(R_{16}^2 S_x^2 + S_y^2(1-\rho^2)\right).$$

Jeelani et al. (2013) suggested an estimator with the use of coefficient of skweness and quartile deviation of the auxiliary information in the simple random sampling for the estimation of the population mean as follows:

$$\hat{\hat{Y}}_{17} = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}\beta_1+QD)}(\bar{X}\beta_1 + QD).$$

The bias, related constant and the MSE for Jeelani et al. (2013) estimator is given by

$$B\left(\hat{\hat{Y}}_{17}\right) = \frac{(1-f)}{n}\frac{S_x^2}{\bar{Y}}R_{17}^2, \qquad R_{17} = \frac{\bar{Y}\beta_1}{(\bar{X}\beta_1+QD)} \quad MSE\left(\hat{\hat{Y}}_{17}\right) = \frac{(1-f)}{n}\left(R_{17}^2 S_x^2 + S_y^2(1-\rho^2)\right).$$

3. Proposed Modified Ratio Estimators

Motivated by the mentioned estimators in Section 2, we propose some new modified ratio type estimators. It is relevant to note that the measures like range, variance, standard deviation and mean deviation are affected by extreme values in the population, whereas the Gini's mean difference, Downton's method and probability weighted moments measures are robust and are more effective in the presence of outliers in the population. The proposed estimators using the linear combination of population coefficient of variation, population coefficient of correlation, Gini's mean difference estimator, Downton's method and probability weighted moments can be formulated as follows:

$$\hat{\hat{Y}}_{p1} = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}+G)}(\bar{X} + G),$$

$$\hat{\hat{Y}}_{p2} = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}\rho+G)}(\bar{X}\rho + G),$$

$$\hat{\hat{Y}}_{p3} = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}C_x+G)}(\bar{X}C_x + G),$$

$$\hat{\hat{Y}}_{p4} = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}+D)}(\bar{X} + D),$$

$$\hat{\hat{Y}}_{p5} = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}\rho+D)}(\bar{X}\rho + D),$$

$$\hat{\hat{Y}}_{p6} = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}C_x+D)}(\bar{X}C_x + D),$$

$$\hat{\hat{Y}}_{p7} = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}+S_{pw})}\left(\bar{X} + S_{pw}\right),$$

$$\hat{\bar{Y}}_{p8} = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}\rho+S_{pw})}(\bar{X}\rho+S_{pw}),$$

$$\hat{\bar{Y}}_{p9} = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}C_x+S_{pw})}(\bar{X}C_x+S_{pw}).$$

The biases, related constants and the MSE for suggested estimators can be obtained as follows:

$$B\left(\hat{\bar{Y}}_{p1}\right) = \frac{(1-f)}{n}\frac{S_x^2}{\bar{Y}}R_{p1}^2, \quad R_{p1} = \frac{\bar{Y}}{(\bar{X}+G)} \qquad MSE\left(\hat{\bar{Y}}_{p1}\right) = \frac{(1-f)}{n}\left(R_{p1}^2S_x^2+S_y^2(1-\rho^2)\right),$$

$$B\left(\hat{\bar{Y}}_{p2}\right) = \frac{(1-f)}{n}\frac{S_x^2}{\bar{Y}}R_{p2}^2, \quad R_{p2} = \frac{\bar{Y}\rho}{(\bar{X}\rho+G)} \qquad MSE\left(\hat{\bar{Y}}_{p2}\right) = \frac{(1-f)}{n}\left(R_{p2}^2S_x^2+S_y^2(1-\rho^2)\right),$$

$$B\left(\hat{\bar{Y}}_{p3}\right) = \frac{(1-f)}{n}\frac{S_x^2}{\bar{Y}}R_{p3}^2, \quad R_{p3} = \frac{\bar{Y}C_x}{(\bar{X}C_x+G)} \qquad MSE\left(\hat{\bar{Y}}_{p3}\right) = \frac{(1-f)}{n}\left(R_{p3}^2S_x^2+S_y^2(1-\rho^2)\right),$$

$$B\left(\hat{\bar{Y}}_{p4}\right) = \frac{(1-f)}{n}\frac{S_x^2}{\bar{Y}}R_{p4}^2, \quad R_{p4} = \frac{\bar{Y}}{(\bar{X}+D)} \qquad MSE\left(\hat{\bar{Y}}_{p4}\right) = \frac{(1-f)}{n}\left(R_{p4}^2S_x^2+S_y^2(1-\rho^2)\right),$$

$$B\left(\hat{\bar{Y}}_{p5}\right) = \frac{(1-f)}{n}\frac{S_x^2}{\bar{Y}}R_{p5}^2, \quad R_{p5} = \frac{\bar{Y}\rho}{(\bar{X}\rho+D)} \qquad MSE\left(\hat{\bar{Y}}_{p5}\right) = \frac{(1-f)}{n}\left(R_{p5}^2S_x^2+S_y^2(1-\rho^2)\right),$$

$$B\left(\hat{\bar{Y}}_{p6}\right) = \frac{(1-f)}{n}\frac{S_x^2}{\bar{Y}}R_{p6}^2, \quad R_{p6} = \frac{\bar{Y}C_x}{(\bar{X}C_x+D)} \qquad MSE\left(\hat{\bar{Y}}_{p6}\right) = \frac{(1-f)}{n}\left(R_{p6}^2S_x^2+S_y^2(1-\rho^2)\right),$$

$$B\left(\hat{\bar{Y}}_{p7}\right) = \frac{(1-f)}{n}\frac{S_x^2}{\bar{Y}}R_{p7}^2, \quad R_{p7} = \frac{\bar{Y}}{(\bar{X}+S_{pw})} \qquad MSE\left(\hat{\bar{Y}}_{p7}\right) = \frac{(1-f)}{n}\left(R_{p7}^2S_x^2+S_y^2(1-\rho^2)\right),$$

$$B\left(\hat{\bar{Y}}_{p8}\right) = \frac{(1-f)}{n}\frac{S_x^2}{\bar{Y}}R_{p8}^2, \quad R_{p8} = \frac{\bar{Y}\rho}{(\bar{X}\rho+S_{pw})} \qquad MSE\left(\hat{\bar{Y}}_{p8}\right) = \frac{(1-f)}{n}\left(R_{p8}^2S_x^2+S_y^2(1-\rho^2)\right),$$

$$B\left(\hat{\bar{Y}}_{p9}\right) = \frac{(1-f)}{n}\frac{S_x^2}{\bar{Y}}R_{p9}^2, \quad R_{p9} = \frac{\bar{Y}C_x}{(\bar{X}C_x+S_{pw})} \qquad MSE\left(\hat{\bar{Y}}_{p9}\right) = \frac{(1-f)}{n}\left(R_{p9}^2S_x^2+S_y^2(1-\rho^2)\right).$$

4. Efficiency Comparisons

In this section, the efficiency conditions for the proposed ratio estimators have been derived algebraically according to usual ratio estimator and existing ratio estimators in literature.

4.1. Comparison with usual ratio estimator

The proposed ratio estimators are more efficient than that of the usual ratio estimator if

$$MSE\left(\hat{\bar{Y}}_{pj}\right) \le MSE\left(\hat{\bar{Y}}_{r}\right),$$

$$\frac{(1-f)}{n}\left(R_{pj}^2S_x^2+S_y^2(1-\rho^2)\right) \le \frac{(1-f)}{n}\left(S_y^2+R^2S_x^2-2R\rho S_xS_y\right),$$

$$R_{pj}^2S_x^2-\rho^2S_y^2-R^2S_x^2+2R\rho S_xS_y \le 0,$$

$$\left(\rho S_y-RS_x\right)^2-R_{pj}^2S_x^2 \ge 0,$$

$$\left(\rho S_y-RS_x+R_{pj}S_x\right)\left(\rho S_y-RS_x-R_{pj}S_x\right) \ge 0.$$

Condition I: $\left(\rho S_y - RS_x + R_{pj}S_x\right) \leq 0$ and $\left(\rho S_y - RS_x - R_{pj}S_x\right) \leq 0$

After solving the condition I, we get

$$\left(\frac{\rho S_y - RS_x}{S_x}\right) \leq R_{pj} \leq \left(\frac{RS_x - \rho S_y}{S_x}\right)$$

Condition II: $\left(\rho S_y - RS_x + R_{pj}S_x\right) \geq 0$ and $\left(\rho S_y - RS_x - R_{pj}S_x\right) \geq 0$

After simplifying the Condition II, we get

$$\left(\frac{RS_x - \rho S_y}{S_x}\right) \leq R_{pj} \leq \left(\frac{\rho S_y - RS_x}{S_x}\right).$$

Hence,

$$MSE\left(\hat{\bar{Y}}_{pj}\right) \leq MSE\left(\hat{\bar{Y}}_r\right),$$

$$\left(\frac{\rho S_y - RS_x}{S_x}\right) \leq R_{pj} \leq \left(\frac{RS_x - \rho S_y}{S_x}\right),$$

or

$$\left(\frac{RS_x - \rho S_y}{S_x}\right) \leq R_{pj} \leq \left(\frac{\rho S_y - RS_x}{S_x}\right). \qquad \text{where } j = 1,2,\ldots,9.$$

4.2. Comparisons with existing ratio estimators

From the expressions of the MSE of the proposed estimators and the existing estimators, we have derived the conditions for which the proposed estimators are more efficient than the existing modified ratio estimators as follows:

$$MSE\left(\hat{\bar{Y}}_{pj}\right) \leq MSE\left(\hat{\bar{Y}}_i\right),$$

$$\frac{(1-f)}{n}\left(R_{pj}^2 S_x^2 + S_y^2(1-\rho^2)\right) \leq \frac{(1-f)}{n}\left(R_i^2 S_x^2 + S_y^2(1-\rho^2)\right),$$

$$R_{pj}^2 S_x^2 \leq R_i^2 S_x^2,$$

$$R_{pj} \leq R_i,$$

where $j = 1, 2, \ldots, 9$ and $i = 1, 2, \ldots, 17$.

5. Empirical Study

The performances of the suggested ratio estimators are evaluated and compared with the usual ratio estimator and the mentioned ratio estimators in Section 2 by using 4 natural populations. The Population 1 is taken from page 177 of Singh and Chaudhary (1986), Populations 2 and 3 are taken from page 228 of Murthy (1967), and the Population 4 is taken from Kadilar and Cingi (2004).

The statistics of 4 populations are given as follows:

Population 1: Singh and Chaudhary (1986)

$N = 34$	$n = 20$	$\bar{Y} = 856.4117$	$\bar{X} = 199.4412$
$\rho = 0.4453$	$S_y = 733.1407$	$C_y = 0.8561$	$S_x = 150.2150$
$C_x = 0.7531$	$\beta_2 = 1.0445$	$\beta_1 = 1.1823$	$M_d = 142.50$
$QD = 80.25$	$G = 162.996$	$D = 144.481$	$S_{pw} = 142.990$

Population 2: Murthy (1967)

$N = 80$	$n = 20$	$\bar{Y} = 5182.637$	$\bar{X} = 285.125$
$\rho = 0.915$	$S_y = 1835.659$	$C_y = 0.3540$	$S_x = 279.429$
$C_x = 0.948$	$\beta_2 = 1.301$	$\beta_1 = 0.698$	$M_d = 148.00$
$QD = 179.375$	$G = 279.711$	$D = 247.938$	$S_{pw} = 244.838$

Population 3: Murthy (1967)

$N = 80$	$n = 20$	$\bar{Y} = 5182.637$	$\bar{X} = 1126.463$
$\rho = 0.941$	$S_y = 1835.659$	$C_y = 0.3540$	$S_x = 845.610$
$C_x = 0.751$	$\beta_2 = -0.063$	$\beta_1 = 1.050$	$M_d = 757.500$
$QD = 588.125$	$G = 904.081$	$D = 801.381$	$S_{pw} = 791.364$

Population 4: Kadilar and Cingi (2004)

$N = 106$	$n = 40$	$\bar{Y} = 2212.59$	$\bar{X} = 27421.70$
$\rho = 0.860$	$S_y = 11551.53$	$C_y = 5.22$	$S_x = 57460.61$
$C_x = 2.10$	$\beta_2 = 34.572$	$\beta_1 = 2.122$	$M_d = 7297.50$
$QD = 12156.25$	$G = 40201.69$	$D = 35634.990$	$S_{pw} = 35298.810$

The values of the related constants and the biases of the existing and proposed modified ratio estimators are given in Table 1, whereas the values of the MSE of the existing and proposed estimators are given in Table 2.

Table 1: The related constants and biases of the existing and the proposed ratio estimators

Estimator	Constant				Bias			
	Population 1	Population 2	Population 3	Population 4	Population 1	Population 2	Population 3	Population 4
$\hat{\bar{Y}}_r$	4.294	18.177	4.601	0.0807	4.94	115.09	60.88	171.32
$\hat{\bar{Y}}_1$	4.294	18.177	4.601	0.0807	10.00	174.83	109.52	151.20
$\hat{\bar{Y}}_2$	4.294	18.177	4.601	0.0807	9.93	173.67	109.37	151.18
$\hat{\bar{Y}}_3$	4.278	18.116	4.598	0.0806	9.89	173.98	109.53	150.82
$\hat{\bar{Y}}_4$	4.272	18.132	4.601	0.0807	9.93	173.18	111.86	151.20
$\hat{\bar{Y}}_5$	4.279	18.090	4.650	0.0806	9.87	173.93	109.53	151.02
$\hat{\bar{Y}}_6$	4.264	18.130	4.601	0.0807	9.96	173.71	109.34	151.19
$\hat{\bar{Y}}_7$	4.285	18.119	4.597	0.0807	9.94	173.65	109.27	151.20
$\hat{\bar{Y}}_8$	4.281	18.115	4.596	0.0807	9.83	173.57	109.36	151.17
$\hat{\bar{Y}}_9$	4.258	18.111	4.598	0.8067	9.96	173.23	112.46	151.20
$\hat{\bar{Y}}_{10}$	4.285	18.094	4.662	0.0806	9.77	173.90	109.53	150.76
$\hat{\bar{Y}}_{11}$	4.244	18.128	4.601	0.0807	9.89	173.24	109.31	151.14
$\hat{\bar{Y}}_{12}$	4.269	18.094	4.597	0.0807	9.91	174.17	109.53	151.13
$\hat{\bar{Y}}_{13}$	4.275	18.143	4.601	0.0637	3.40	75.76	39.15	94.32
$\hat{\bar{Y}}_{14}$	2.504	11.966	2.751	0.0715	2.63	73.02	30.47	119.04
$\hat{\bar{Y}}_{15}$	2.204	11.747	2.427	0.0767	3.89	89.31	40.69	136.64
$\hat{\bar{Y}}_{16}$	2.676	12.991	2.804	0.0801	3.53	57.48	1.186	148.10
$\hat{\bar{Y}}_{17}$	2.550	10.422	0.478	0.0742	5.26	79.42	48.85	128.08
$\hat{\bar{Y}}_{p1}$	**2.363**	**9.175**	**2.552**	**0.0327**	**3.03**	**44.55**	**33.71**	**24.87**
$\hat{\bar{Y}}_{p2}$	**2.059**	**8.935**	**2.224**	**0.0475**	**2.30**	**42.25**	**25.58**	**52.34**
$\hat{\bar{Y}}_{p3}$	**1.515**	**8.772**	**2.483**	**0.0297**	**1.24**	**40.72**	**31.91**	**20.59**
$\hat{\bar{Y}}_{p4}$	**1.635**	**9.320**	**2.620**	**0.0320**	**1.45**	**45.96**	**35.53**	**23.85**
$\hat{\bar{Y}}_{p5}$	**2.189**	**9.483**	**2.362**	**0.0498**	**2.60**	**47.58**	**28.87**	**57.60**
$\hat{\bar{Y}}_{p6}$	**2.490**	**9.722**	**2.688**	**0.0351**	**3.36**	**50.02**	**37.39**	**28.59**
$\hat{\bar{Y}}_{p7}$	**1.645**	**9.377**	**2.635**	**0.0322**	**1.47**	**46.53**	**35.91**	**24.12**
$\hat{\bar{Y}}_{p8}$	**2.200**	**9.540**	**2.377**	**0.0500**	**2.63**	**48.16**	**29.22**	**58.02**
$\hat{\bar{Y}}_{p9}$	**2.501**	**9.779**	**2.702**	**0.0353**	**3.39**	**50.61**	**37.78**	**28.90**

Table 2: The MSE values of the existing and the proposed ratio estimators

Estimator	Mean square Error			
	Population 1	**Population 2**	**Population 3**	**Population 4**
$\hat{\bar{Y}}_r$	10960.76	413243.60	189775.10	984589.70
$\hat{\bar{Y}}_1$	17437.65	926660.70	581994.20	889617.50
$\hat{\bar{Y}}_2$	17373.31	920662.50	581238.50	889566.40
$\hat{\bar{Y}}_3$	17348.62	922242.50	582058.10	888775.70
$\hat{\bar{Y}}_4$	17376.04	918082.10	594119.80	889616.00
$\hat{\bar{Y}}_5$	17319.75	922003.40	582079.30	889215.30
$\hat{\bar{Y}}_6$	17399.52	920873.20	581046.80	889596.60
$\hat{\bar{Y}}_7$	17387.08	920560.30	580732.70	889607.50
$\hat{\bar{Y}}_8$	17294.19	920108.20	581191.40	889557.80
$\hat{\bar{Y}}_9$	17401.14	918382.80	597260.90	889616.90
$\hat{\bar{Y}}_{10}$	17239.66	921833.60	582062.10	888634.40
$\hat{\bar{Y}}_{11}$	17336.98	918450.90	580937.60	889492.50
$\hat{\bar{Y}}_{12}$	17362.26	923260.70	582055.10	889452.90
$\hat{\bar{Y}}_{13}$	11785.70	413230.80	217319.80	763783.60
$\hat{\bar{Y}}_{14}$	11127.47	399044.90	172323.80	818477.40
$\hat{\bar{Y}}_{15}$	12199.76	483450.40	225319.50	857402.20
$\hat{\bar{Y}}_{16}$	11892.07	318486.70	20545.47	884526.80
$\hat{\bar{Y}}_{17}$	13376.04	432164.60	267595.20	838466.80
$\hat{\bar{Y}}_{p1}$	**11465.43**	**251457.80**	**189080.30**	**610126.10**
$\hat{\bar{Y}}_{p2}$	**10841.88**	**239515.60**	**146971.80**	**670914.00**
$\hat{\bar{Y}}_{p3}$	**9937.20**	**231591.30**	**179770.50**	**600579.70**
$\hat{\bar{Y}}_{p4}$	**10113.06**	**258768.30**	**198518.70**	**607875.10**
$\hat{\bar{Y}}_{p5}$	**11097.24**	**267178.40**	**164020.70**	**682552.70**
$\hat{\bar{Y}}_{p6}$	**11752.23**	**279802.00**	**208187.20**	**618381.50**
$\hat{\bar{Y}}_{p7}$	**10129.01**	**261697.40**	**200516.20**	**608480.30**
$\hat{\bar{Y}}_{p8}$	**11119.84**	**270154.60**	**165857.40**	**683478.00**
$\hat{\bar{Y}}_{p9}$	**11777.26**	**282843.60**	**210216.90**	**619061.50**

It can be observed that the related constants and the biases of the suggested modified ratio estimators are smaller than the usual ratio estimator and the existing ratio estimators in literature. From Table 2, it is obvious that the proposed estimators perform better than the usual ratio estimator and the existing modified ratio estimators in terms of MSE, which indicates that the proposed estimators are more efficient.

To get more insight into the proposals of the study, we have also evaluated the percentage relative efficiencies (PRE) of the proposed estimators (p) with respect to the existing estimators (e), which is computed by the formula given below

$$PRE(e,p) = \frac{MSE(e)}{MSE(p)} \times 100$$

and the values of PRE are given in Tables 3 -6.

Table 3: PRE of the proposed estimators and the existing estimators for population 1

Existing Estimators	Proposed Estimators								
	$\hat{\bar{Y}}_{p1}$	$\hat{\bar{Y}}_{p2}$	$\hat{\bar{Y}}_{p3}$	$\hat{\bar{Y}}_{p4}$	$\hat{\bar{Y}}_{p5}$	$\hat{\bar{Y}}_{p6}$	$\hat{\bar{Y}}_{p7}$	$\hat{\bar{Y}}_{p8}$	$\hat{\bar{Y}}_{p9}$
$\hat{\bar{Y}}_{r}$	95.6	101.1	110.3	108.4	98.8	93.3	108.2	98.6	93.1
$\hat{\bar{Y}}_{1}$	152.1	160.8	175.5	172.4	157.1	148.4	172.2	156.8	148.1
$\hat{\bar{Y}}_{2}$	151.5	160.2	174.8	171.8	156.6	147.8	171.5	156.2	147.5
$\hat{\bar{Y}}_{3}$	151.3	160.0	174.6	171.5	156.3	147.6	171.3	156.0	147.3
$\hat{\bar{Y}}_{4}$	151.6	160.3	174.9	171.8	156.6	147.9	171.5	156.3	147.5
$\hat{\bar{Y}}_{5}$	151.1	159.7	174.3	171.3	156.1	147.4	171.0	155.8	147.1
$\hat{\bar{Y}}_{6}$	151.8	160.5	175.1	172.1	156.8	148.1	171.8	156.5	147.7
$\hat{\bar{Y}}_{7}$	151.6	160.4	175.0	171.9	156.7	147.9	171.7	156.4	147.6
$\hat{\bar{Y}}_{8}$	150.8	159.5	174.0	171.0	155.8	147.2	170.7	155.5	146.8
$\hat{\bar{Y}}_{9}$	151.8	160.5	175.1	172.1	156.8	148.1	171.8	156.5	147.8
$\hat{\bar{Y}}_{10}$	150.4	159.0	173.5	170.5	155.4	146.7	170.2	155.0	146.4
$\hat{\bar{Y}}_{11}$	151.2	159.9	174.5	171.4	156.2	147.5	171.2	155.9	147.2
$\hat{\bar{Y}}_{12}$	151.4	160.1	174.7	171.7	156.5	147.7	171.4	156.1	147.4
$\hat{\bar{Y}}_{13}$	102.8	108.7	118.6	116.5	106.2	100.3	116.4	106.0	100.1
$\hat{\bar{Y}}_{14}$	97.1	102.6	112.0	110.0	100.3	94.7	109.9	100.1	94.5
$\hat{\bar{Y}}_{15}$	106.4	112.5	122.8	120.6	109.9	103.8	120.4	109.7	103.6
$\hat{\bar{Y}}_{16}$	103.7	109.7	119.7	117.6	107.2	101.2	117.4	106.9	101.0
$\hat{\bar{Y}}_{17}$	116.7	123.4	134.6	132.3	120.5	113.8	132.1	120.3	113.6

Table 4: PRE of the proposed estimators and the existing estimators for population 2

Existing Estimators	Proposed Estimators								
	$\hat{\bar{Y}}_{p1}$	$\hat{\bar{Y}}_{p2}$	$\hat{\bar{Y}}_{p3}$	$\hat{\bar{Y}}_{p4}$	$\hat{\bar{Y}}_{p5}$	$\hat{\bar{Y}}_{p6}$	$\hat{\bar{Y}}_{p7}$	$\hat{\bar{Y}}_{p8}$	$\hat{\bar{Y}}_{p9}$
$\hat{\bar{Y}}_{r}$	164.3	172.5	178.4	159.7	154.7	147.7	157.9	153.0	146.1
$\hat{\bar{Y}}_{1}$	368.5	386.9	400.1	358.1	346.8	331.2	354.1	343.0	327.6
$\hat{\bar{Y}}_{2}$	366.1	384.4	397.5	355.8	344.6	329.0	351.8	340.8	325.5
$\hat{\bar{Y}}_{3}$	366.8	385.0	398.2	356.4	345.2	329.6	352.4	341.4	326.1
$\hat{\bar{Y}}_{4}$	365.1	383.3	396.4	354.8	343.6	328.1	350.8	339.8	324.6
$\hat{\bar{Y}}_{5}$	366.7	384.9	398.1	356.3	345.1	329.5	352.3	341.3	326.0
$\hat{\bar{Y}}_{6}$	366.2	384.5	397.6	355.9	344.7	329.1	351.9	340.9	325.6
$\hat{\bar{Y}}_{7}$	366.1	384.3	397.5	355.7	344.5	329.0	351.8	340.8	325.5
$\hat{\bar{Y}}_{8}$	365.9	384.2	397.3	355.6	344.4	328.8	351.6	340.6	325.3
$\hat{\bar{Y}}_{9}$	365.2	383.4	396.6	354.9	343.7	328.2	350.9	339.9	324.7
$\hat{\bar{Y}}_{10}$	366.6	384.9	398.0	356.2	345.0	329.5	352.3	341.2	325.9
$\hat{\bar{Y}}_{11}$	365.3	383.5	396.6	354.9	343.8	328.3	351.0	340.0	324.7
$\hat{\bar{Y}}_{12}$	367.2	385.5	398.7	356.8	345.6	330.0	352.8	341.8	326.4
$\hat{\bar{Y}}_{13}$	164.3	172.5	178.4	159.7	154.7	147.7	157.9	153.0	146.1
$\hat{\bar{Y}}_{14}$	158.7	166.6	172.3	154.2	149.4	142.6	152.5	147.7	141.1
$\hat{\bar{Y}}_{15}$	192.3	201.8	208.8	186.8	180.9	172.8	184.7	179.0	170.9
$\hat{\bar{Y}}_{16}$	126.7	133.0	137.5	123.1	119.2	113.8	121.7	117.9	112.6
$\hat{\bar{Y}}_{17}$	171.9	180.4	186.6	167.0	161.8	154.5	165.1	160.0	152.8

Table 5: PRE of the proposed estimators and the existing estimators for population 3

Existing Estimators	Proposed Estimators								
	$\hat{\bar{Y}}_{p1}$	$\hat{\bar{Y}}_{p2}$	$\hat{\bar{Y}}_{p3}$	$\hat{\bar{Y}}_{p4}$	$\hat{\bar{Y}}_{p5}$	$\hat{\bar{Y}}_{p6}$	$\hat{\bar{Y}}_{p7}$	$\hat{\bar{Y}}_{p8}$	$\hat{\bar{Y}}_{p9}$
$\hat{\bar{Y}}_r$	100.4	129.1	105.6	95.6	115.7	91.2	94.6	114.4	90.3
$\hat{\bar{Y}}_1$	307.8	396.0	323.7	293.2	354.8	279.6	290.2	350.9	276.9
$\hat{\bar{Y}}_2$	307.4	395.5	323.3	292.8	354.4	279.2	289.9	350.4	276.5
$\hat{\bar{Y}}_3$	307.8	396.0	323.8	293.2	354.9	279.6	290.3	350.9	276.9
$\hat{\bar{Y}}_4$	314.2	404.2	330.5	299.3	362.2	285.4	296.3	358.2	282.6
$\hat{\bar{Y}}_5$	307.8	396.0	323.8	293.2	354.9	279.6	290.3	351.0	276.9
$\hat{\bar{Y}}_6$	307.3	395.3	323.2	292.7	354.3	279.1	289.8	350.3	276.4
$\hat{\bar{Y}}_7$	307.1	395.1	323.0	292.5	354.1	278.9	289.6	350.1	276.3
$\hat{\bar{Y}}_8$	307.4	395.4	323.3	292.8	354.3	279.2	289.8	350.4	276.5
$\hat{\bar{Y}}_9$	315.9	406.4	332.2	300.9	364.1	286.9	297.9	360.1	284.1
$\hat{\bar{Y}}_{10}$	307.8	396.0	323.8	293.2	354.9	279.6	290.3	350.9	276.9
$\hat{\bar{Y}}_{11}$	307.2	395.3	323.2	292.6	354.2	279.0	289.7	350.3	276.4
$\hat{\bar{Y}}_{12}$	307.8	396.0	323.8	293.2	354.9	279.6	290.3	350.9	276.9
$\hat{\bar{Y}}_{13}$	114.9	147.9	120.9	109.5	132.5	104.4	108.4	131.0	103.4
$\hat{\bar{Y}}_{14}$	91.1	117.2	95.9	86.8	105.1	82.8	85.9	103.9	82.0
$\hat{\bar{Y}}_{15}$	119.2	153.3	125.3	113.5	137.4	108.2	112.4	135.9	107.2
$\hat{\bar{Y}}_{16}$	10.9	14.0	11.4	10.3	12.5	9.9	10.2	12.4	9.8
$\hat{\bar{Y}}_{17}$	141.5	182.1	148.9	134.8	163.1	128.5	133.5	161.3	127.3

Table 6: PRE of the proposed estimators and the existing estimators for population 4

Existing Estimators	Proposed Estimators								
	$\hat{\bar{Y}}_{p1}$	$\hat{\bar{Y}}_{p2}$	$\hat{\bar{Y}}_{p3}$	$\hat{\bar{Y}}_{p4}$	$\hat{\bar{Y}}_{p5}$	$\hat{\bar{Y}}_{p6}$	$\hat{\bar{Y}}_{p7}$	$\hat{\bar{Y}}_{p8}$	$\hat{\bar{Y}}_{p9}$
$\hat{\bar{Y}}_r$	146.1	131.9	149.3	162.0	144.3	159.2	161.8	144.1	159.0
$\hat{\bar{Y}}_1$	132.0	119.2	134.9	146.3	130.3	143.9	146.2	130.2	143.7
$\hat{\bar{Y}}_2$	132.0	119.2	134.9	146.3	130.3	143.9	146.2	130.2	143.7
$\hat{\bar{Y}}_3$	131.8	119.1	134.8	146.2	130.2	143.7	146.1	130.0	143.6
$\hat{\bar{Y}}_4$	132.0	119.2	134.9	146.3	130.3	143.9	146.2	130.2	143.7
$\hat{\bar{Y}}_5$	131.9	119.2	134.8	146.3	130.3	143.8	146.1	130.1	143.6
$\hat{\bar{Y}}_6$	132.0	119.2	134.9	146.3	130.3	143.9	146.2	130.2	143.7
$\hat{\bar{Y}}_7$	132.0	119.2	134.9	146.3	130.3	143.9	146.2	130.2	143.7
$\hat{\bar{Y}}_8$	132.0	119.2	134.9	146.3	130.3	143.9	146.2	130.2	143.7
$\hat{\bar{Y}}_9$	132.0	119.2	134.9	146.3	130.3	143.9	146.2	130.2	143.7
$\hat{\bar{Y}}_{10}$	131.8	119.1	134.7	146.2	130.2	143.7	146.0	130.0	143.5
$\hat{\bar{Y}}_{11}$	131.9	119.2	134.9	146.3	130.3	143.8	146.2	130.1	143.7
$\hat{\bar{Y}}_{12}$	131.9	119.2	134.9	146.3	130.3	143.8	146.2	130.1	143.7
$\hat{\bar{Y}}_{13}$	113.3	102.3	115.8	125.6	111.9	123.5	125.5	111.7	123.4
$\hat{\bar{Y}}_{14}$	121.4	109.7	124.1	134.6	119.9	132.4	134.5	119.8	132.2
$\hat{\bar{Y}}_{15}$	127.2	114.9	130.0	141.0	125.6	138.7	140.9	125.4	138.5
$\hat{\bar{Y}}_{16}$	131.2	118.5	134.1	145.5	129.6	143.0	145.4	129.4	142.9
$\hat{\bar{Y}}_{17}$	124.4	112.4	127.1	137.9	122.8	135.6	137.8	122.7	135.4

The proposed estimators perform well as compared to the usual ratio estimator and the existing estimators in terms of PRE (cf. Tables 3-6). This shows that the suggested ratio type estimators are more efficient than the usual ratio estimator and the existing ratio estimators.

5. Conclusion

Availability of the auxiliary information is helpful to improve the sampling plan or to enhance the estimation of the properties of the variables of interest. Some new modified linear regression type ratio estimators using known values of the population coefficient of variation, population correlation coefficient, the Gini's mean difference, Downton's method and probability weighted moments based measures are suggested. The proposed estimators are more efficient than that of the existing estimators in terms of constant, bias and mean square error under different populations. Hence we strongly recommend the use of the proposed modified linear regression type ratio estimators for practical concerns and the estimation of the population mean and may be preferred over the usual ratio estimator and the existing modified linear regression type ratio estimators when unusual observations are present in the auxiliary variables.

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