A Class of Estimators of Population Mean in Case of Post Stratification

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Abstract

This paper proposes a class of ratio-cum-product type estimators in case of post-stratification. Particular members of the proposed class of ratio-cum-product type estimators have been identified and studied thoroughly from efficiency point of view. It has been shown that the identified particular estimators are more efficient than the usual unbiased estimator, Ige and Tripathi (1989) estimators, Chouhan (2012) estimators, Tailor et al. (2016) estimator and other considered estimators. An empirical study has been carried out to demonstrate the performance of the proposed estimators.

Keywords: Mean squared error, Bias, Ratio-cum-product estimator.

1. Introduction

Cochran (1940) and Robson (1957) envisaged classical ratio and product estimators which were studied in case of post stratification by Ige and Tripathi (1989). Recently, Lone and Tailor (2014) and Lone and Tailor (2015) proposed ratio and product type exponential estimators in case of post-stratification. Chouhan (2012) proposed class of ratio type estimators using various known parameters of auxiliary variates in case of post stratification. Tailor et al. (2015) proposed dual to Ige and Tripathi (1989) ratio and product estimators. Tailor et al. (2016) proposed a ratio-cum-product type estimator in case of post-stratification. Singh (1967) used information on population mean of two auxiliary variates and proposed ratio-cum-product type estimator for population mean in simple random sampling. Singh (1967) and Chouhan (2012) motivate authors to propose the class of ratio-cum-product type estimators in case of post-stratification. Many Researchers including Holt and Smith (1979), Jagers et al. (1985), Jagers (1986), Ige and Tripathi (1989), Agrawal and Panday (1993), Singh and Ruiz Espejo (2003) Tailor et al. (2011) and Tailor et al. (2015) contributed well in case of post-stratification.

Let us consider a finite population $U=(U_1,U_2,...,U_N)$ of size N which is divided into L strata of size $N_1,N_2,...,N_L$ such that $\sum_{h=1}^{L} N_h = N$. Let y be the study variate and x is the auxiliary variate correlated positively with the study variate y and z is the auxiliary variate, negatively correlated with the study variate y. Let y_{hi} be the observation on i^{th} unit of h^{th} stratum for study variate y and x_{hi} and z_{hi} be the observation on i^{th} unit of h^{th} stratum for auxiliary variates x and z respectively, then

$\bar{X}_h = \frac{1}{N_h}\sum_{i=1}^{N_h} x_{hi}$: h^{th} stratum mean for the auxiliary variate x,

$\bar{Y}_h = \frac{1}{N_h}\sum_{i=1}^{N_h} y_{hi}$: h^{th} stratum mean for the study variate y,

$\bar{Z}_h = \frac{1}{N_h}\sum_{i=1}^{N_h} x_{hi}$: h^{th} stratum mean for the auxiliary variate z,

$\bar{X} = \frac{1}{N}\sum_{h=1}^{L}\sum_{i=1}^{N_h} x_{hi} = \sum_{h=1}^{L} W_h \bar{X}_h$: Population mean of the auxiliary variate x,

$\bar{Y} = \frac{1}{N}\sum_{h=1}^{L}\sum_{i=1}^{N_h} y_{hi} = \sum_{h=1}^{L} W_h \bar{Y}_h$: Population mean of the study variate y and

$\bar{Z} = \sum_{h=1}^{L} W_h \bar{Z}_h$: Population mean of the auxiliary variate z.

A sample of size n is drawn from population U using simple random sampling without replacement. After selecting the sample, it is observed that which units belong to h^{th} stratum. Let n_h be the size of the sample falling in h^{th} stratum such that $\sum_{h=1}^{L} n_h = n$ Here it is assumed that n is so large that possibility of n_h being zero is very small.

In case of post-stratification, usual unbiased estimator of population mean $\bar{Y}$ is defined as

$$\bar{y}_{PS} = \sum_{h=1}^{L} W_h \bar{y}_h , \tag{1.1}$$

where

$W_h = \frac{N_h}{N}$ is the weight of the h^{th} stratum and $\bar{y}_h = \frac{1}{n_h}\sum_{i=1}^{n_h} y_{hi}$ is sample mean of n_h sample units that fall in the h^{th} stratum.

Using the results from Stephen (1945), the variance of $\bar{y}_{PS}$ to the first degree of approximation is obtained as

$$Var(\bar{y}_{PS}) = \left(\frac{1}{n} - \frac{1}{N}\right)\sum_{h=1}^{L} W_h S_{yh}^2 + \frac{1}{n^2}\sum_{h=1}^{L}(1 - W_h)S_{yh}^2 \tag{1.2}$$

where $S_{yh}^2 = \frac{1}{N_h - 1}\sum_{i=1}^{N_h}\left(y_{hi} - \bar{Y}_h\right)^2$.

Ige and Tripathi (1989) defined a ratio and a product type estimator in case of post-stratification as

$$\hat{\bar{Y}}_{PS}^{R} = \bar{y}_{PS}\left(\frac{\bar{X}}{\bar{x}_{PS}}\right) \tag{1.3}$$

and

$$\hat{\bar{Y}}_{PS}^{P} = \bar{y}_{PS}\left(\frac{\bar{z}_{PS}}{\bar{Z}}\right). \tag{1.4}$$

where $\bar{x}_{PS} = \sum_{h=1}^{L} W_h \bar{x}_h$ and $\bar{z}_{PS} = \sum_{h=1}^{L} W_h \bar{z}_h$ are the unbiased estimators of population means in case of post-stratification and $\bar{x}_h$ and $\bar{z}_h$ are the means of the samples of size n_h that in case of post-stratification and $\bar{x}_h$ and $\bar{z}_h$ are the means of the samples of size fall in h^{th} stratum.

Mean squared error of the Ige and Tripathi (1989) estimators $\hat{\bar{Y}}_{PS}^{R}$ and $\hat{\bar{Y}}_{PS}^{P}$ are

$$MSE\left(\hat{\bar{Y}}_{PS}^{R}\right) = \left(\frac{1}{n} - \frac{1}{N}\right)\sum_{h=1}^{L} W_h \left(S_{yh}^2 + R_1^2 S_{xh}^2 - 2R_1 S_{yxh}\right) \tag{1.5}$$

and

$$MSE\left(\hat{\bar{Y}}_{PS}^{P}\right) = \left(\frac{1}{n} - \frac{1}{N}\right)\sum_{h=1}^{L} W_h \left(S_{yh}^2 + R_2^2 S_{zh}^2 + 2R_2 S_{yzh}\right). \tag{1.6}$$

where

$$R_1 = \frac{\bar{Y}}{\bar{X}} \quad \text{and} \quad R_2 = \frac{\bar{Y}}{\bar{Z}}.$$

Chouhan (2012) proposed the following ratio type estimators for population mean $\bar{Y}$ in case of post-stratification as

$$\hat{\bar{Y}}_{PS}^{RSD} = \bar{y}_{ps}\left(\frac{\sum_{h=1}^{L} W_h\left(\bar{X}_h + C_{xh}\right)}{\sum_{h=1}^{L} W_h\left(\bar{x}_h + C_{xh}\right)}\right), \tag{1.7}$$

$$\hat{\bar{Y}}_{PS}^{RSE} = \bar{y}_{ps}\left(\frac{\sum_{h=1}^{L} W_h\left(\bar{X}_h + \beta_{2h}(x)\right)}{\sum_{h=1}^{L} W_h\left(\bar{x}_h + \beta_{2h}(x)\right)}\right) \tag{1.8}$$

and

$$\hat{\bar{Y}}_{PS}^{RST} = \bar{y}_{ps}\left(\frac{\sum_{h=1}^{L} W_h\left(\bar{X}_h + \rho_{yxh}\right)}{\sum_{h=1}^{L} W_h\left(\bar{x}_h + \rho_{yxh}\right)}\right), \tag{1.9}$$

Mean squared errors of the estimators $\hat{\bar{Y}}_{PS}^{RSD}$, $\hat{\bar{Y}}_{PS}^{RSE}$ and $\hat{\bar{Y}}_{PS}^{RST}$ upto the first degree of approximation are obtained as

$$MSE\left(\hat{\bar{Y}}_{PS}^{RSD}\right) = \left(\frac{1}{n} - \frac{1}{N}\right)\sum_{h=1}^{L} W_h\left(S_{yh}^2 + R_{n1}^2 S_{xh}^2 - 2R_{n1} S_{yxh}\right), \tag{1.10}$$

$$MSE\left(\hat{\bar{Y}}_{PS}^{RSE}\right) = \left(\frac{1}{n} - \frac{1}{N}\right)\sum_{h=1}^{L} W_h\left(S_{yh}^2 + R_{n2}^2 S_{xh}^2 - 2R_{n2} S_{yxh}\right), \tag{1.11}$$

$$MSE\left(\hat{\bar{Y}}_{PS}^{RST}\right) = \left(\frac{1}{n} - \frac{1}{N}\right)\sum_{h=1}^{L} W_h\left(S_{yh}^2 + R_{n3}^2 S_{xh}^2 - 2R_{n3} S_{yxh}\right), \tag{1.12}$$

Where as the product version of $\hat{\bar{Y}}_{PS}^{RSD}$, $\hat{\bar{Y}}_{PS}^{RSE}$ and $\hat{\bar{Y}}_{PS}^{RST}$ can be written as

$$\hat{\bar{Y}}_{PS}^{PSD} = \bar{y}_{ps}\left(\frac{\sum_{h=1}^{L} W_h\left(\bar{z}_h + C_{zh}\right)}{\sum_{h=1}^{L} W_h\left(\bar{Z}_h + C_{zh}\right)}\right), \tag{1.13}$$

$$\hat{\bar{Y}}_{PS}^{PSE} = \bar{y}_{ps}\left(\frac{\sum_{h=1}^{L} W_h\left(\bar{z}_h + \beta_{2h}(z)\right)}{\sum_{h=1}^{L} W_h\left(\bar{Z}_h + \beta_{2h}(z)\right)}\right) \tag{1.14}$$

and

$$\hat{\bar{Y}}_{PS}^{PST} = \bar{y}_{ps}\left(\frac{\sum_{h=1}^{L} W_h\left(\bar{z}_h + \rho_{yzh}\right)}{\sum_{h=1}^{L} W_h\left(\bar{Z}_h + \rho_{yzh}\right)}\right) \tag{1.15}$$

Upto the first degree of approximation, mean squared error of the estimators $\hat{\bar{Y}}_{PS}^{PSD}$, $\hat{\bar{Y}}_{PS}^{PSE}$ and $\hat{\bar{Y}}_{PS}^{PST}$ are obtained as

$$MSE\left(\hat{\bar{Y}}_{PS}^{PSD}\right) = \left(\frac{1}{n} - \frac{1}{N}\right)\sum_{h=1}^{L} W_h\left(S_{yh}^2 + R_{m1}^2 S_{zh}^2 + 2R_{m1} S_{yzh}\right), \tag{1.16}$$

$$MSE\left(\hat{\bar{Y}}_{PS}^{PSE}\right) = \left(\frac{1}{n} - \frac{1}{N}\right)\sum_{h=1}^{L} W_h\left(S_{yh}^2 + R_{m2}^2 S_{zh}^2 + 2R_{m2} S_{yzh}\right), \tag{1.17}$$

$$MSE\left(\hat{\bar{Y}}_{PS}^{PST}\right)=\left(\frac{1}{n}-\frac{1}{N}\right)\sum_{h=1}^{L}W_h\left(S_{yh}^2+R_{m3}^2S_{zh}^2+2R_{m3}\,S_{yzh}\right), \tag{1.18}$$

where

$$R_{ni}=\begin{cases}\bar{Y}\Big/\sum_{h=1}^{L}W_h\left(\bar{X}_h+C_{xh}\right) & i=1\\ \bar{Y}\Big/\sum_{h=1}^{L}W_h\left(\bar{X}_h+\beta_{2h}(x)\right) & i=2\\ \bar{Y}\Big/\sum_{h=1}^{L}W_h\left(\bar{X}_h+\rho_{yxh}\right) & i=3\end{cases}\quad and\ R_{mi}=\begin{cases}\bar{Y}\Big/\sum_{h=1}^{L}W_h\left(\bar{Z}_h+C_{xh}\right) & i=1\\ \bar{Y}\Big/\sum_{h=1}^{L}W_h\left(\bar{Z}_h+\beta_{2h}(z)\right) & i=2\\ \bar{Y}\Big/\sum_{h=1}^{L}W_h\left(\bar{Z}_h+\rho_{yzh}\right) & i=3\end{cases}.$$

Tailor et al. (2016) proposed a ratio-cum-product type estimator in case of post-stratification as

$$\hat{\bar{Y}}_{PS}^{RP}=\bar{y}_{ps}\left(\frac{\sum_{h=1}^{L}W_h\bar{X}_h}{\sum_{h=1}^{L}W_h\bar{x}_h}\right)\left(\frac{\sum_{h=1}^{L}W_h\bar{z}_h}{\sum_{h=1}^{L}W_h\bar{Z}_h}\right) \tag{1.19}$$

Up to the first degree of approximation, mean squared error of $\hat{\bar{Y}}_{PS}^{RP}$ is obtained as

$$MSE\left(\hat{\bar{Y}}_{PS}^{RP}\right)=\left(\frac{1}{n}-\frac{1}{N}\right)\sum_{h=1}^{L}W_h\left[S_{yh}^2+R_1^2S_{xh}^2+R_2^2S_{zh}^2-2\left(R_1S_{yxh}+R_1\,R_2S_{xzh}-R_2S_{yzh}\right)\right] \tag{1.20}$$

2. Proposed Class of Ratio-Cum- Product Type Estimators

In the line of Singh (1967), we suggest a class of ratio-cum-product type estimators for estimating population mean $\bar{Y}$ as

$$t=\bar{y}_{ps}\left(\frac{\sum_{h=1}^{L}W_h\left(a_h\bar{X}_h+b_h\right)}{\sum_{h=1}^{L}W_h\left(a_h\bar{x}_h+b_h\right)}\right)\left(\frac{\sum_{h=1}^{L}W_h\left(c_h\bar{z}_h+d_h\right)}{\sum_{h=1}^{L}W_h\left(c_h\bar{Z}_h+d_h\right)}\right) \tag{2.1}$$

where $\left(a_h, b_h, c_h \text{ and } d_h\right)$ are the function of population parameters of the auxiliary variates.

To obtain the bias and mean squared error of the proposed estimator t, we write

$\bar{y}_h=\bar{Y}_h\left(1+e_{0h}\right)$, $\bar{x}_h=\bar{X}_h\left(1+e_{1h}\right)$ and $\bar{z}_h=\bar{Z}_h\left(1+e_{2h}\right)$ such that

$$E(e_{0h})=E(e_{1h})=E(e_{2h})=0,$$

$$E(e_{0h}^2)=\left(\frac{1}{nW_h}-\frac{1}{N_h}\right)C_{yh}^2,$$

$$E(e_{1h}^2)=\left(\frac{1}{nW_h}-\frac{1}{N_h}\right)C_{xh}^2,$$

$$E(e_{2h}^2)=\left(\frac{1}{nW_h}-\frac{1}{N_h}\right)C_{zh}^2,$$

$$E(e_{0h}e_{1h})=\left(\frac{1}{nW_h}-\frac{1}{N_h}\right)\rho_{yxh}C_{yh}C_{xh}\ ,$$

$$E(e_{0h}e_{2h})=\left(\frac{1}{nW_h}-\frac{1}{N_h}\right)\rho_{yzh}C_{yh}C_{zh}\quad \text{and}$$

$$E(e_{1h}e_{2h})=\left(\frac{1}{nW_h}-\frac{1}{N_h}\right)\rho_{xzh}C_{xh}C_{zh}.$$

Expressing (2.1) in terms of e's, we have

$$t=\bar{Y}\left(1+\frac{\sum_{h=1}^{L}W_h\bar{Y}_h e_{0h}}{\bar{Y}}\right)\left(1+\frac{\sum_{h=1}^{L}W_h a_h\bar{X}_h e_{1h}}{X_n}\right)^{-1}\left(1+\frac{\sum_{h=1}^{L}W_h c_h\bar{Z}_h e_{2h}}{X_m}\right)$$

$$t=\bar{Y}\,(1+e_0)(1+e_1)^{-1}(1+e_2)$$

$$\Rightarrow t-\bar{Y}=\bar{Y}\left(e_0-e_1+e_2+e_1^2-e_1e_2-e_0e_1+e_0e_2\right) \tag{2.2}$$

where $e_0=\dfrac{\sum_{h=1}^{L}W_h\bar{Y}_h e_{0h}}{\bar{Y}}$, $e_1=\dfrac{\sum_{h=1}^{L}W_h a_h\bar{X}_h e_{1h}}{X_n}$ and $e_2=\dfrac{\sum_{h=1}^{L}W_h c_h\bar{Z}_h e_{2h}}{X_m}$.

Taking expectation on both sides of (2.2), we get the bias of the proposed estimator t upto the first degree of approximation as

$$B(t)=\left(\frac{1}{n}-\frac{1}{N}\right)\left[\frac{1}{X_n}\left\{\sum_{h=1}^{L}W_h S_{xh}^2\,a_h^2 R_n-\sum_{h=1}^{L}W_h\,a_h S_{yxh}\right\}\right.$$
$$\left.+\frac{1}{X_m}\left\{\sum_{h=1}^{L}W_h c_h S_{yzh}-\sum_{h=1}^{L}W_h R_n a_h c_h S_{xzh}\right\}\right] \tag{2.3}$$

Squaring and taking expectation on both sides of (2.2), the mean squared error of the proposed estimator t, upto the first degree of approximation is obtained as

$$MSE(t)=\left(\frac{1}{n}-\frac{1}{N}\right)\sum_{h=1}^{L}W_h\left[S_{yh}^2+R_n^2a_h^2S_{xh}^2+R_m^2c_h^2S_{zh}^2-2R_na_hS_{yxh}\right.$$
$$\left.-2R_n\,R_ma_hc_hS_{xzh}+2c_hR_mS_{yzh}\right] \tag{2.4}$$

where $R_n=\dfrac{\overline{Y}}{X_n}$, $R_m=\dfrac{\overline{Y}}{X_m}$, $X_n=\sum_{h=1}^{L}W_h\left(a_h\overline{X}_h+b_h\right)$ and $X_m=\sum_{h=1}^{L}W_h\left(c_h\overline{Z}_h+d_h\right)$.

Equation (2.4) can also be written as

$$MSE(t)=A+R_n^2B+R_m^2C-2R_nD-2R_n\,R_mE+2R_mF \tag{2.5}$$

where

$$A=\left(\frac{1}{n}-\frac{1}{N}\right)\sum_{h=1}^{L}W_hS_{yh}^2\ ,$$

$$B=\left(\frac{1}{n}-\frac{1}{N}\right)\sum_{h=1}^{L}W_ha_h^2S_{xh}^2\ ,$$

$$C=\left(\frac{1}{n}-\frac{1}{N}\right)\sum_{h=1}^{L}W_hc_h^2S_{zh}^2\ ,$$

$$D=\left(\frac{1}{n}-\frac{1}{N}\right)\sum_{h=1}^{L}W_ha_hS_{yxh}\ ,$$

$$E=\left(\frac{1}{n}-\frac{1}{N}\right)\sum_{h=1}^{L}W_ha_hc_hS_{xzh}\quad\text{and}$$

$$F=\left(\frac{1}{n}-\frac{1}{N}\right)\sum_{h=1}^{L}W_hc_hS_{yzh}\ .$$

The MSE of t is minimum when

$$\begin{cases}R_n=\dfrac{DC-EF}{BC-E^2}=R_n^*(say)\\[2ex] R_m=\dfrac{DE-BF}{BC-E^2}=R_m^*(say)\end{cases} \tag{2.6}$$

Putting (2.6) in (2.5), we get the minimum mean squared error of the proposed estimator t as

$$MSE_{\min}(t)=A\left[1-\rho\right] \tag{2.7}$$

where $\rho=\dfrac{(CD^2+BF^2-2DEF)}{A(BC-E^2)}$.

Table 2.1: Some Known Members of t

Generated estimators	Values of Constant			
	a_h	b_h	c_h	d_h
$\hat{\bar{Y}}_{PS}^{R} = \bar{y}_{PS}$ **Usual unbiased estimator**	0	1	0	1
$\hat{\bar{Y}}_{PS}^{R} = \bar{y}_{PS}\left(\frac{\bar{X}}{\bar{x}_{PS}}\right)$	1	0	0	1
$\hat{\bar{Y}}_{PS}^{P} = \bar{y}_{PS}\left(\frac{\bar{z}_{PS}}{\bar{Z}}\right)$ **Ige and Trapthi (1987) estimators**	0	1	1	0
$\hat{\bar{Y}}_{PS}^{RSD} = \bar{y}_{ps}\left(\frac{\sum_{h=1}^{L} W_h\left(\bar{X}_h + C_{xh}\right)}{\sum_{h=1}^{L} W_h\left(\bar{x}_h + C_{xh}\right)}\right)$	1	C_{xh}	0	1
$\hat{\bar{Y}}_{PS}^{RSE} = \bar{y}_{ps}\left(\frac{\sum_{h=1}^{L} W_h\left(\bar{X}_h + \beta_{2h}(x)\right)}{\sum_{h=1}^{L} W_h\left(\bar{x}_h + \beta_{2h}(x)\right)}\right)$	1	$\beta_{2h}(x)$	0	1
$\hat{\bar{Y}}_{PS}^{RST} = \bar{y}_{ps}\left(\frac{\sum_{h=1}^{L} W_h\left(\bar{X}_h + \rho_{yxh}\right)}{\sum_{h=1}^{L} W_h\left(\bar{x}_h + \rho_{yxh}\right)}\right)$ **Chouhan (2012) estimators**	1	ρ_{yxh}	0	1
$\hat{\bar{Y}}_{PS}^{RP} = \bar{y}_{ps}\left(\frac{\sum_{h=1}^{L} W_h \bar{X}_h}{\sum_{h=1}^{L} W_h \bar{x}_h}\right)\left(\frac{\sum_{h=1}^{L} W_h \bar{z}_h}{\sum_{h=1}^{L} W_h \bar{Z}_h}\right)$ **Tailor et al. (2016) estimator**	1	0	1	0
$t_1 = \bar{y}_{ps}\left(\frac{\sum_{h=1}^{L} W_h\left(\bar{X}_h + C_{xh}\right)}{\sum_{h=1}^{L} W_h\left(\bar{x}_h + C_{xh}\right)}\right) \times \left(\frac{\sum_{h=1}^{L} W_h\left(\bar{z}_h + C_{zh}\right)}{\sum_{h=1}^{L} W_h\left(\bar{Z}_h + C_{zh}\right)}\right)$	1	C_{xh}	1	C_{zh}

$t_2 = \bar{y}_{ps}\left(\dfrac{\sum_{h=1}^{L} W_h\left(\bar{X}_h + \beta_{2h}(x)\right)}{\sum_{h=1}^{L} W_h\left(\bar{x}_h + \beta_{2h}(x)\right)}\right) \times \left(\dfrac{\sum_{h=1}^{L} W_h\left(\bar{z}_h + \beta_{2h}(z)\right)}{\sum_{h=1}^{L} W_h\left(\bar{Z}_h + \beta_{2h}(z)\right)}\right)$	1	$\beta_{2h}(x)$	1	$\beta_{2h}(z)$
$t_3 = \bar{y}_{ps}\left(\dfrac{\sum_{h=1}^{L} W_h\left(\bar{X}_h + \rho_{yxh}\right)}{\sum_{h=1}^{L} W_h\left(\bar{x}_h + \rho_{yxh}\right)}\right) \times \left(\dfrac{\sum_{h=1}^{L} W_h\left(\bar{z}_h + \rho_{yzh}\right)}{\sum_{h=1}^{L} W_h\left(\bar{Z}_h + \rho_{yzh}\right)}\right)$	1	ρ_{yxh}	1	ρ_{yzh}

3. Efficiency Comparisons

From (1.2), (1.5), (1.6), (1.10), (1.11), (1.12), (1.16) (1.17), (1.18), (1.20) and (2.7), we conclude that the proposed class of ratio-cum- product type estimators t would be more efficient than

(i) $\bar{y}_{PS}$ if

$$\rho > 0, \tag{3.1}$$

(ii) $\hat{\bar{Y}}_{PS}^{R}$ if

$$\rho \sum_{h=1}^{L} W_h S_{yh}^2 \sum_{h=1}^{L} W_h \left(R_1^2 S_{xh}^2 - 2R_1 S_{yxh}\right) > 0, \tag{3.2}$$

(iii) $\hat{\bar{Y}}_{PS}^{P}$ if

$$\rho \sum_{h=1}^{L} W_h S_{yh}^2 + \sum_{h=1}^{L} W_h \left(R_1^2 S_{zh}^2 + 2R_2 S_{yzh}\right) > 0, \tag{3.3}$$

(iv) $\hat{\bar{Y}}_{PS}^{RSD}$ if

$$\rho\sum_{h=1}^{L}W_h S_{yh}^2+\sum_{h=1}^{L}W_h\left(R_{n1}^2 S_{xh}^2-2R_{n1}S_{yxh}\right)>0, \tag{3.4}$$

(v) $\hat{\bar{Y}}_{PS}^{RSE}$ if

$$\rho\sum_{h=1}^{L}W_h S_{yh}^2+\sum_{h=1}^{L}W_h\left(R_{n2}^2 S_{xh}^2-2R_{n2}S_{yxh}\right)>0, \tag{3.5}$$

(vi) $\hat{\bar{Y}}_{PS}^{RST}$ if

$$\rho\sum_{h=1}^{L}W_h S_{yh}^2+\sum_{h=1}^{L}W_h\left(R_{n3}^2 S_{xh}^2-2R_{n3}S_{yxh}\right)>0, \tag{3.6}$$

(vii) $\hat{\bar{Y}}_{PS}^{PSD}$ if

$$\rho\sum_{h=1}^{L}W_h S_{yh}^2+\sum_{h=1}^{L}W_h\left(R_{m1}^2 S_{zh}^2+2R_{m1}S_{yzh}\right)>0, \tag{3.7}$$

(viii) $\hat{\bar{Y}}_{PS}^{PSE}$ if

$$\rho\sum_{h=1}^{L}W_h S_{yh}^2+\sum_{h=1}^{L}W_h\left(R_{m2}^2 S_{zh}^2+2R_{m2}S_{yzh}\right)>0, \tag{3.8}$$

(ix) $\hat{\bar{Y}}_{PS}^{PST}$ if

$$\rho\sum_{h=1}^{L}W_h S_{yh}^2+\sum_{h=1}^{L}W_h\left(R_{m3}^2 S_{zh}^2+2R_{m3}S_{yzh}\right)>0, \tag{3.9}$$

(x) $\hat{\bar{Y}}_{PS}^{RP}$ if

$$\rho\sum_{h=1}^{L}W_h S_{yh}^2+\sum_{h=1}^{L}W_h\left[R_1^2 S_{xh}^2+R_2^2 S_{zh}^2-2\left(R_1 S_{yxh}+R_1\,R_2 S_{xzh}-R_2 S_{yzh}\right)\right]>0, \tag{3.10}$$

Expressions (3.1) and (3.10) are the conditions under which the proposed class of ratio-cum-product type estimator t is better than usual unbiased estimator $\bar{y}_{PS}$, Ige and Tripathi (1989) estimators $\hat{\bar{Y}}_{PS}^{R}$ and $\hat{\bar{Y}}_{PS}^{P}$, Chouhan (2012) estimators $\hat{\bar{Y}}_{PS}^{RSD}$, $\hat{\bar{Y}}_{PS}^{RSE}$ and $\hat{\bar{Y}}_{PS}^{RST}$, Tailor et al. (2016) estimator $\hat{\bar{Y}}_{PS}^{RP}$ and other considered estimators $\hat{\bar{Y}}_{PS}^{PSD}$, $\hat{\bar{Y}}_{PS}^{PSE}$ and $\hat{\bar{Y}}_{PS}^{PST}$.

4. Study on the particular members of the proposed class of ratio-cum- product type estimators t

To illustrate the general result, we have considered the new members t_1, t_2 and t_3 of the proposed class of ratio-cum- product type estimators t defined in table 6.2.1 as

$$t_1 = \bar{y}_{ps}\left(\frac{\sum_{h=1}^{L} W_h\left(\bar{X}_h + C_{xh}\right)}{\sum_{h=1}^{L} W_h\left(\bar{x}_h + C_{xh}\right)}\right)\left(\frac{\sum_{h=1}^{L} W_h\left(\bar{z}_h + C_{zh}\right)}{\sum_{h=1}^{L} W_h\left(\bar{Z}_h + C_{zh}\right)}\right), \tag{4.1}$$

$$t_2 = \bar{y}_{ps}\left(\frac{\sum_{h=1}^{L} W_h\left(\bar{X}_h + \beta_{2h}(x)\right)}{\sum_{h=1}^{L} W_h\left(\bar{x}_h + \beta_{2h}(x)\right)}\right)\left(\frac{\sum_{h=1}^{L} W_h\left(\bar{z}_h + \beta_{2h}(z)\right)}{\sum_{h=1}^{L} W_h\left(\bar{Z}_h + \beta_{2h}(z)\right)}\right) \tag{4.2}$$

and

$$t_3 = \bar{y}_{ps}\left(\frac{\sum_{h=1}^{L} W_h\left(\bar{X}_h + \rho_{yxh}\right)}{\sum_{h=1}^{L} W_h\left(\bar{x}_h + \rho_{yxh}\right)}\right)\left(\frac{\sum_{h=1}^{L} W_h\left(\bar{z}_h + \rho_{yzh}\right)}{\sum_{h=1}^{L} W_h\left(\bar{Z}_h + \rho_{yzh}\right)}\right) \tag{4.3}$$

Upto the first degree of approximation, the mean squared errors of the estimators t_1, t_2 and t_3 are obtained as

$$MSE(t_1) = \left(\frac{1}{n} - \frac{1}{N}\right)\sum_{h=1}^{L} W_h\left[S_{yh}^2 + R_{n1}^2 S_{xh}^2 + R_{m1}^2 S_{zh}^2 - 2R_{n1} S_{yxh} - 2R_{n1} R_{m1} S_{xzh} + 2R_{m1} S_{yzh}\right] \tag{4.4}$$

$$MSE(t_2) = \left(\frac{1}{n} - \frac{1}{N}\right)\sum_{h=1}^{L} W_h\left[S_{yh}^2 + R_{n2}^2 S_{xh}^2 + R_{m2}^2 S_{zh}^2 - 2R_{n2} S_{yxh} - 2R_{n2} R_{m2} S_{xzh} + 2R_{m2} S_{yzh}\right] \tag{4.5}$$

and

$$MSE(t_3) = \left(\frac{1}{n} - \frac{1}{N}\right)\sum_{h=1}^{L} W_h\left[S_{yh}^2 + R_{n3}^2 S_{xh}^2 + R_{m3}^2 S_{zh}^2 - 2R_{n3} S_{yxh} - 2R_{n3} R_{m3} S_{xzh} + 2R_{m3} S_{yzh}\right] \tag{4.6}$$

5. Empirical Study

To exhibit the performance of the proposed estimators, two population data sets are being considered. Descriptions of data sets are given by

Table 5.1: Population I- [Source: Chouhan (2012)]

y : Productivity (MT/Hectare)

x : Production in ‘000 Tons and

z : Area in '000 Hectare

Constant	Stratum I	Stratum II
N_h	10	10
n_h	4	4
$\bar{Y}_h$	1.70	3.67
$\bar{X}_h$	10.41	289.14
$\bar{Z}_h$	6.32	80.67
S_{yh}	0.50	1.41
S_{xh}	3.53	111.61
S_{zh}	1.19	10.82
S_{yxh}	1.60	144.87
S_{yzh}	-0.05	-7.04
S_{xzh}	1.38	-92.02
$\beta_{2h}(x)$	1.97	2.90
$\beta_{2h}(z)$	4.12	3.66

Table 5.2: Population II- [Source: Murthy (1967), p 228]

z : Number of workers
y : Output and
x : Fixed capital

Constant	Stratum I	Stratum II
N_h	5	5
n_h	2	2
$\bar{Y}_h$	1925.8	315.6
$\bar{X}_h$	214.4	333.8
$\bar{Z}_h$	51.80	60.60
S_{yh}	615.92	340.38
S_{xh}	74.87	66.35
S_{zh}	0.75	4.84
S_{yxh}	39360.68	22356.50
S_{yzh}	411.16	1536.24
S_{xzh}	38.08	287.92
$\beta_{2h}(x)$	1.88	2.32
$\beta_{2h}(z)$	1.84	1.49

Table 5.3: Percent Relative Efficiencies of the estimators $\bar{y}_{PS}$, $\hat{\bar{Y}}_{PS}^{R}$, $\hat{\bar{Y}}_{PS}^{P}$, $\hat{\bar{Y}}_{PS}^{RSD}$, $\hat{\bar{Y}}_{PS}^{RSE}$, $\hat{\bar{Y}}_{PS}^{RST}$, $\hat{\bar{Y}}_{PS}^{PSD}$, $\hat{\bar{Y}}_{PS}^{PSE}$, $\hat{\bar{Y}}_{PS}^{PST}$, $\hat{\bar{Y}}_{PS}^{RP}$, t_1, t_2 and t_3 with respect to $\bar{y}_{PS}$.

Estimators	Percent Relative Efficiencies	
	Population I	**Population II**
$\bar{y}_{PS}$	100.00	100.00
$\hat{\bar{Y}}_{PS}^{R}$	223.74	313.75
$\hat{\bar{Y}}_{PS}^{P}$	123.31	85.02
$\hat{\bar{Y}}_{PS}^{RSD}$	225.20	376.69
$\hat{\bar{Y}}_{PS}^{RSE}$	233.80	384.14
$\hat{\bar{Y}}_{PS}^{RST}$	227.49	378.73
$\hat{\bar{Y}}_{PS}^{PSD}$	123.36	85.03
$\hat{\bar{Y}}_{PS}^{PSE}$	123.38	85.43
$\hat{\bar{Y}}_{PS}^{PST}$	123.47	85.25
$\hat{\bar{Y}}_{PS}^{RP}$	288.16	258.07
t_1	**291.07**	**404.85**
t_2	**312.91**	**409.54**
t_3	**294.31**	**405.42**

6. Conclusion

A class of ratio-cum-product type estimators for population mean has been defined. The usual unbiased estimator, Ige and Tripathi (1989) estimators, Chouhan (2012) estimators and Tailor et al. (2016) estimator have been identified to be the member of the proposed class of ratio-cum-product type estimators t. Section 3 provides the conditions under which the proposed estimator t has less mean squared error as compared to the mean squared error of the other considered estimators. It is observed that particular members t_1, t_2 and t_3 have higher percent relative efficiencies in comparison to usual unbiased estimator $\bar{y}_{PS}$, Ige and Tripathi (1989) estimators $\hat{\bar{Y}}_{PS}^{R}$ and $\hat{\bar{Y}}_{PS}^{P}$, Chouhan (2012) estimators $\hat{\bar{Y}}_{PS}^{RSD}$, $\hat{\bar{Y}}_{PS}^{RSE}$ and $\hat{\bar{Y}}_{PS}^{RST}$, Tailor et al. (2016) estimator $\hat{\bar{Y}}_{PS}^{RP}$ and other estimators $\hat{\bar{Y}}_{PS}^{PSD}$, $\hat{\bar{Y}}_{PS}^{PSE}$ and $\hat{\bar{Y}}_{PS}^{PST}$. Hence, it can be concluded that the proposed members t_1, t_2 and t_3 are recommended for use in practice. Thus, it has been concluded that there is an enough scope of generating better estimators from the proposed class of ratio-cum-product type estimator t using suitable known parameters of auxiliary variates.

Acknowledgment

The authors are grateful to the reviewers for their constructive comments and valuable suggestions regarding improvement of the article.

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