Chance Constrained Compromise Mixed Allocation in Multivariate Stratified Sampling

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Abstract

Consider a multivariate stratified population with L strata and $p > 1$ characteristics. Let the estimation of the population means be of interest. In such cases the traditional individual optimum allocations may differ widely from characteristic to characteristic and there will be no obvious compromise between them unless they are highly correlated. As a result there does not exist a single set of allocations $(n_1, n_2, ..., n_L)$ that can be practically implemented on all characteristics. Assuming the characteristics independent many authors worked out allocations based on different compromise criterion such allocations are called compromise allocation. These allocations are optimum for all characteristics in some sense. Ahsan et al. (2005) introduced the concept of 'Mixed allocation' in univariate stratified sampling. Later on Varshney et al. (2011) extended it for multivariate case and called it a 'Compromise Mixed Allocation'. Ahsan et al. (2013) worked on mixed allocation in stratified sampling by using the 'Chance Constrained Programming Technique', that allows the cost constraint to be violated by a specified small probability. This paper presents a more realistic approach to the compromise mixed allocation by formulating the problem as a Chance Constrained Nonlinear Programming Problem in which the per unit measurement costs in various strata are random variables. The application of this approach is exhibited through a numerical example assuming normal distributions of the random parameters.

Keywords: Multivariate stratified surveys, Compromise mixed allocation, Chance constrained programming.

Mathematics Subject Classifications 62D05 62H12

1. Introduction

In stratified sampling, the use of any particular type of allocation depends on the nature of the population, objectives of survey, the available budget, etc. Practically, there are situations where all strata of a stratified population do not allow the use of a single type of allocation. For example, in the absence of the strata weight W_h, strata variance S_h^2 and per unit measurement cost $c_h; h = 1, 2, ..., L$, optimum allocation can not be used. Similarly, if strata sizes $N_h; h = 1, 2, ..., L$ are unknown proportional allocation can not be used. Thus, when no information about some strata of the population is available, 'equal' allocation may be used for a given total sample size for that strata. If the only information available for some strata is N_h, 'proportional' allocation may be used, that is, $n_h \propto N_h$. If S_h^2 for some strata are available, an allocation $n_h \propto W_h S_h$ may be used. When there is evidence to believe that the relative standard error of the stratum mean $\bar{Y}_h$ based on one

sample unit does not vary considerably over some strata, an allocation $n_h \propto W_h \bar{Y}_h$ may be used. As the range R_h of a stratum provides an approximation to the standard deviation in the absence of the knowledge of the stratum standard deviations S_h, the allocation may be taken as $n_h \propto W_h R_h$ (See Murthy (1967)). When full information about the population is available, the obvious choice is the 'optimum allocation'. Ahsan et al. (2005) divided the strata into disjoint groups and used different allocations for different groups. They called their allocation as 'Mixed Allocation'. Later on Varshney and Ahsan (2011) extended the work of Ahsan et al. (2005) for multivariate stratified sampling. Ahsan et al. (2013) worked out mixed allocation using chance constraint that allows the cost constraint to be violated by a specified small probability.

In this paper the work of Ahsan and Naz (2013) is extended for multivariate stratified sampling, where in cost constraint, a small probability of violation is allowed. Because per unit cost of measurement $c_h; h = 1, 2, ..., L$ may vary during the course of the survey due to random causes in practice it becomes a random variable. Thus the problem of compromise mixed allocation can be viewed as a Chance Constrained Nonlinear Programming Problem (CCNLPP).

In section 2 and 3 the work of Varshney et al. (2011) and Ahsan et al. (2013) are summarized for the sake of continuity.

2. Compromise mixed allocation

Let the L strata of a multivariate stratified population be divided into k groups $G_1, G_2, ..., G_k$, where the group $G_j; j = 1, 2, ..., k$ consists of L_j strata with $\sum_{j=1}^{k} L_j = L$. The strata are grouped according to the information available for them, that is, full information partial information or zero information (See Kozak (2006(a), 2006(b))). Or according to some other measure of information as desired in the 'Introduction' of this manuscript.

Without loss of generality we can assume that the first L_1 strata constitute the group G_1, the next L_2 strata constitute the group G_2, and so on. The last L_k strata will constitute the group G_k. In order to use different types of allocations in different groups define

$$n_h = \alpha_j \beta_h \; ; h \in I_j \; ; j = 1, 2, ..., k \tag{2.1}$$

where $I_j; j = 1, 2, ..., k$ is the set of indices of the strata constituting the group G_j, $\beta_h; h \in I_j; j = 1, 2, ..., k$ are known constants depending upon the type of allocation to be used in the group G_j and $\alpha_j; j = 1, 2, ..., k$ are the decision variables to be determined. For example if in any particular group, say G_p, equal allocation is to be used then $\beta_h = 1; h \in I_p$, proportional allocation in the qth group G_q may be applied by assuming

$\beta_h = W_h; h \in I_q$, for optimum allocation in the rth group G_r, β_h may be taken as $\beta_h = W_h S_h / \sqrt{c_h}$; $h \in I_r$ and so on.

It is to be noted that under the above scheme

$$I_j = \left\{ \sum_{i=1}^{j-1} L_i + 1, \sum_{i=1}^{j-1} L_i + 2,..., \sum_{i=1}^{j} L_i \right\}; j = 1, 2,..., k \tag{2.2}$$

where

$$I_r \bigcap I_s = \phi; \quad r \neq s \quad \text{and} \quad \bigcup_{j=1}^{k} I_j = \{1, 2,..., L\}$$

Using compromise criterion of Yates (1960), Varshney and Ahsan (2011) formulated the problem of finding a compromise mixed allocation as the following Nonlinear Programming Problem (NLPP) for multivariate stratified sampling as

$$\text{Minimize} \quad \sum_{l=1}^{p} a_l \sum_{j=1}^{k} \sum_{h \in I_j} \frac{W_h^2 S_{lh}^2}{\alpha_j \beta_h} \tag{2.3}$$

$$\text{subject to} \quad \sum_{j=1}^{k} \sum_{h \in I_j} \alpha_j c_h \beta_h \leq C_0 \tag{2.4}$$

$$\text{and} \quad \frac{2}{\beta_h} \leq \alpha_j \leq \frac{N_h}{\beta_h}; h \in I_j; j = 1, 2,..., k \tag{2.5}$$

where $a_l > 0$; $l = 1, 2,..., p$ are weights associated to the lth characteristics according to its relative importance, c_h is the cost of measuring all the p characteristics on a selected unit of the hth stratum, that is, $c_h = \sum_{l=1}^{p} c_{lh}$; $h = 1, 2,..., L$, c_{lh} denote the per unit cost of measuring the lth characteristic in the hth stratum. Without loss of generality we can assume that $\sum_{l=1}^{p} a_l = 1$.

Using Lagrange multipliers technique Varshney and Ahsan (2011) obtained the values of α_j as

$$\alpha_j = C_0 \frac{\sqrt{\left(\sum_{h \in I_j} \frac{A_h}{\beta_h} \right) \Big/ \left(\sum_{h \in I_j} c_h \beta_h \right)}}{\sum_{j=1}^{k} \sqrt{\left(\sum_{h \in I_j} \frac{A_h}{\beta_h} \right) \left(\sum_{h \in I_j} c_h \beta_h \right)}}; j = 1, 2,..., k \tag{2.6}$$

where $A_h = W_h^2 \sum_{l=1}^{p} a_l S_{lh}^2$; $h = 1, 2,..., L$.

3. Chance constrained mixed allocation

Ahsan and Naz (2013) formulated the mixed allocation that minimizes the variance of the stratified sample mean for a fixed cost as the following CCNLPP

$$\text{Minimize } F(\alpha_j) = \sum_{j=1}^{k} \sum_{h \in I_j} \frac{W_h^2 S_{lh}^2}{\alpha_j \beta_h} \tag{3.1}$$

$$\text{subject to } P\left[\sum_{j=1}^{k} \sum_{h \in I_j} \alpha_j c_h \beta_h \le C_0 \right] \ge p_0 \tag{3.2}$$

$$\text{and} \quad \alpha_j \ge 0; \ j = 1, 2, ..., k \tag{3.3}$$

Where $c_h \sim N(\mu_{c_h}, \sigma_{c_h}^2)$ because in a survey if the costs for enumerating a characteristic in various strata are not known exactly and these are being estimated from sample costs that may be subjected to random variations. They may increase to a level where the cost constraint is violated.

The chance constraint (3.2) may be expressed as

$$P\left[\sum_{j=1}^{k} \sum_{h \in I_j} q_h \alpha_j \le C_0 \right] \ge p_0 \tag{3.4}$$

where $q_h = c_h \beta_h \sim N(\beta_h \mu_{c_h}, \beta_h^2 \sigma_{c_h}^2)$

Define d_0 as

$$d_0 = \sum_{j=1}^{k} \sum_{h \in I_j} q_h \alpha_j \tag{3.5}$$

Since q_h are normally distributed and α_j are unknown constants, d_0 will also be normally distributed with a mean

$$\bar{d}_0 = \sum_{j=1}^{k} \sum_{h \in I_j} \bar{q}_h \alpha_j \tag{3.6}$$

and variance

$$V(d_0) = V\left(\sum_{j=1}^{k} \sum_{h \in I_j} q_h \alpha_j \right) = \sum_{j=1}^{k} \sum_{h \in I_j} \alpha_j^2 V(q_h) \tag{3.7}$$

Using (3.5), (3.6) and (3.7) the constraint (3.4) can now be expressed as

$$P[d_0 \le C_0] \ge p_0$$

$$\Rightarrow \quad P\left[\frac{d_0 - \bar{d}_0}{\sqrt{V(d_0)}} \le \frac{C_0 - \bar{d}_0}{\sqrt{V(d_0)}} \right] \ge p_0$$

$$\Rightarrow \quad P\left[Z \le \frac{C_0 - \bar{d}_0}{\sqrt{V(d_0)}}\right] \ge p_0 \tag{3.8}$$

where $Z \sim N(0,1)$.

Thus the probability of realizing d_0 smaller than or equal to C_0 can be written as

$$P[d_0 \le C_0] = \phi\left(\frac{C_0 - \bar{d}_0}{\sqrt{V(d_0)}}\right) \tag{3.9}$$

where $\phi(x)$ represents the cumulative distribution function of the standard normal variate evaluated at x.

If γ_0 denotes the value of the standard normal variable at which $\phi(\gamma_0) = p_0$ then the constraint (3.8) implies that

$$\phi\left(\frac{C_0 - \bar{d}_0}{\sqrt{V(d_0)}}\right) \ge \phi(\gamma_0) \tag{3.10}$$

Inequality (3.10) will be satisfied if and only if

$$\left(\frac{C_0 - \bar{d}_0}{\sqrt{V(d_0)}}\right) \ge \gamma_0$$

$$\Rightarrow \quad \bar{d}_0 + \gamma_0\sqrt{V(d_0)} - C_0 \le 0$$

(See Rao S. S. (2010)).

Using (3.6) and (3.7), we get

$$\sum_{j=1}^{k}\sum_{h \in I_j}\bar{q}_h \alpha_j + \gamma_0\sqrt{V(d_0)} - C_0 \le 0$$

or $$\sum_{j=1}^{k}\sum_{h \in I_j}\bar{q}_h \alpha_j + \gamma_0\sqrt{\sum_{j=1}^{k}\sum_{h \in I_j}\alpha_j^2 V(q_h)} - C_0 \le 0 \tag{3.11}$$

Inequality (3.11) gives the deterministic equivalent to the linear chance constraint (3.4).

Thus the solution of the stochastic nonlinear programming problem (3.1)-(3.3) can be obtained by solving the equivalent deterministic programming problem

$$\text{Minimize } F(\alpha_j) = \sum_{j=1}^{k}\sum_{h \in I_j}\frac{W_h^2 S_h^2}{\alpha_j \beta_h} \tag{3.12}$$

$$\text{subject to} \quad \sum_{j=1}^{k}\sum_{h\in I_j} \bar{q}_h \alpha_j + \gamma_0 \sqrt{\sum_{j=1}^{k}\sum_{h\in I_j} \alpha_j^2 V(q_h)} - C_0 \leq 0 \tag{3.13}$$

$$\text{and} \quad \alpha_j \geq 0;\ j = 1, 2, ..., k \tag{3.14}$$

Problem (3.12)-(3.14) is a nonlinear programming problem and could be solved by a suitable nonlinear programming technique. Ahsan and Naz (2013) used the optimization software LINGO (2001) to obtain a solution.

4. Chance constrained compromise mixed allocation

Using the results of sections 2 and 3, the deterministic equivalent of the CCNLPP (3.12)-(3.14) for multivariate case may be expressed as the following NLPP:

$$\text{Minimize} \quad \sum_{l=1}^{p} a_l \ \sum_{j=1}^{k}\sum_{h\in I_j} \frac{W_h^2 S_{lh}^2}{\alpha_j \beta_h} \tag{4.1}$$

$$\text{subject to} \quad \sum_{j=1}^{k}\sum_{h\in I_j} \bar{q}_h \alpha_j + \gamma_0 \sqrt{\sum_{j=1}^{k}\sum_{h\in I_j} \alpha_j^2 V(q_h)} - C_0 \leq 0 \tag{4.2}$$

$$\text{and} \quad \frac{2}{\beta_h} \leq \alpha_j \leq \frac{N_h}{\beta_h};\ h \in I_j;\ j = 1, 2, ..., k \tag{4.3}$$

where $S_{lh}^2;\ h = 1, 2, ..., L;\ l = 1, 2, ..., p$ is the stratum variance for the lth characteristic in the hth stratum. Note that the restrictions in (4.3) are equivalent to $2 \leq n_h \leq N_h;\ h = 1, 2, ..., L$, are the usual restrictions to avoid oversampling and to estimate the strata variances.

When the numerical data are available the NLPP (4.1)-(4.3) may be solved using a suitable Nonlinear Programming Technique. In the next section an application of the proposed formulation is given for an artificial data. The solution to the NLPP (4.1)-(4.3) is obtained by the optimization software LINGO (2001).

5. A numerical illustration

Varshney et al. (2011) gave a numerical illustration using an artificial data given in Table 1.

The values of $\hat{\sigma}_{c_h}^2$ are assumed by authors. The available amount C_0 for measurements is 2000 units. This amount is bifurcated in proportion to $\sum_{h=1}^{4} \bar{c}_{h1}$ and $\sum_{h=1}^{4} \bar{c}_{h2}$ approximately as $C_{01} = 1160$ and $C_{02} = 840$ for the first and second characteristics respectively.

Table 1 Data for seven strata and two characteristics

The grouping scheme and values of $\beta_h, h \in I_j; j = 1, 2, ..., k$ are same as given in Varshney et al. (2011).

Table 2 Expected cost with their estimated variances for the two characteristics

The authors assumed that the probability of violation of the cost constraint is 0.01, that is, the cost of constraint should be satisfied with probability 0.99. This gives the value of γ_0 as 2.33 from standard normal area table.

The strata are so numbered that:

(i)   Strata 1, 2 and 3 constitute group G_1 in which equal allocation is to be used, that is

$$\beta_h = 1; h \in I_1 = \{1, 2, 3\} \tag{5.1}$$

(ii)   Strata 4 and 5 constitute group G_2 in which proportional allocation is to be used, that is

$$\beta_h = W_h; h \in I_2 = \{4, 5\} \tag{5.2}$$

(iii) Strata 6 and 7 constitute group G_3 in which optimum allocation is to be used, that is

$$\beta_h = \sqrt{W_h^2 \sum_{l=1}^{p} a_l S_{lh}^2 / c_{lh}}; h \in I_3 = \{6, 7\}; l = 1, 2 \tag{5.3}$$

(See Varshney et al. (2011)).

It can be seen that $I_j; j = 1, 2, 3$ are mutually exclusive and exhaustive.

It is also assumed that both the characteristics are equally important, that is, $a_1 = a_2 = 0.5$.

Using (5.1), (5.2) and (5.3) the values of $\beta_h; h = 1, 2, ..., 7$ are obtained as

$\beta_1 = \beta_2 = \beta_3 = 1$, $\beta_4 = 0.0872$, $\beta_5 = 0.0932$, $\beta_6 = 1.0301$ and

$$\beta_7 = 1.7121 \tag{5.4}$$

Using the values given in the Table 1 and Table 2 and the values of β_h given in (5.4) the NLPP (4.1)-(4.3) may be expressed as

$$\text{Minimize } \phi(\underline{\alpha}) = \left(\frac{5.26270853}{\alpha_1} + \frac{139.4232983}{\alpha_2} + \frac{16.79198771}{\alpha_3} \right) \tag{5.5}$$

$$\text{subject to} \quad [21\alpha_1 + 2.0716\alpha_2 + 35.9825\alpha_3]$$

$$+ 2.33\sqrt{58\alpha_1^2 + 0.55710992\alpha_2^2 + 216.7032762\alpha_3^2} \leq 2000 \tag{5.6}$$

$$\alpha_1 \geq 0$$
$$\alpha_2 \geq 0 \tag{5.7}$$
$$\alpha_3 \geq 0$$

Using optimization software LINGO the solution to the NLPP (5.5)-(5.7) is obtained as:

$$\alpha_1 = 13.60410, \ \alpha_2 = 211.1268 \ \text{and} \ \alpha_3 = 15.97107.$$

Expression (2.1) give the rounded off compromise mixed allocation for the computed values of $\alpha_j; j = 1, 2, 3$ as

$$n_1 = n_2 = n_3 = \alpha_1 = 13.60410 \cong 14$$
$$n_4 = \alpha_2 \beta_4 = 211.1268 \times 0.08720 = 18.41025 \cong 18$$
$$n_5 = \alpha_2 \beta_5 = 211.1268 \times 0.09320 = 19.67701 \cong 20$$
$$n_6 = \alpha_3 \beta_6 = 15.97107 \times 1.0301 = 16.45179 \cong 16$$
$$n_7 = \alpha_3 \beta_7 = 15.97107 \times 1.7121 = 27.34406 \cong 27.$$

The total sample size $n = \sum_{h=1}^{7} n_h = 123$ (5.4)

Under the above (rounded off) compromise mixed allocation the estimated variances $v(\bar{y}_{lst})$ (ignoring fpc) for the two characteristics are $v(\bar{y}_{1st}) = 1.653680948$ and $v(\bar{y}_{2st}) = 2.561467036$ with a trace value 4.215147984.

6. A Comparative study

The allocations compared are

1.   Proportional allocation

2.   Cochran's compromise allocation

3.   Chatterjee's compromise allocation

4.   Sukhatme's compromise allocation

5.   Proposed compromise mixed allocation

6.1 Proportional allocation

With total sample size $n = 123$ as given in (5.4), the rounded off proportional allocation will be

$$n_{1(prop)} = 23, \ n_{2(prop)} = 28, \ n_{3(prop)} = 21, \ n_{4(prop)} = 11, \ n_{5(prop)} = 11, \ n_{6(prop)} = 16,$$
$$n_{7(prop)} = 13$$

with variances ignoring fpc as $V_{prop}(\bar{y}_{1st}) = 2.619337334, \ V_{prop}(\bar{y}_{2st}) = 3.881857529$. The trace value is 6.501194863.

6.2 Cochran's average allocation

Cochran (1977) gave the compromise criteria by averaging the individual optimum allocations $n_{lh}^*; h = 1, 2, \dots, L; \ l = 1, 2, \dots, p$ that are solutions to the following NLPP for all the p characteristics separately.

$$\left.\begin{array}{l} \text{Minimize} \quad V_l = \sum_{h=1}^{L} \frac{W_h^2 s_{lh}^2}{n_{lh}} \\[4mm] \text{subject to} \quad \sum_{h=1}^{L} \hat{c}_{lh} n_{lh} + \gamma_0 \sqrt{V(\hat{c}_{lh}) n_{lh}^2} \le C_{0l} \; ; l = 1, 2, ..., p \\[4mm] 2 \le n_{lh} \le N_h \; ; h = 1, 2, ..., L \end{array}\right\} \qquad (6.1)$$

where C_{0l} denote the amount assigned for measuring the lth characteristics; $l = 1, 2, ..., p$. Let $\underline{n}_l^* = (n_{l1}^*, n_{l2}^*, ..., n_{lL}^*)$ denote the solution to the lth NLPP in (6.1) with V_l^* as the value of the objective function.

Cochran's compromise allocation is then given by

$$n_{h(a)} = \frac{1}{p} \sum_{l=1}^{p} n_{lh}^* \; ; \; h = 1, 2, ..., L \qquad (6.2)$$

where the suffix 'a' stands for 'Average allocation'.

With the available data (6.2) give the rounding off average allocation as $\underline{n}_{(a)} = (12, 13, 10, 16, 17, 15, 24)$.

With variances (ignoring fpc) as $V_a^*(\bar{y}_{1st}) = 1.869742767$ and $V_a^*(\bar{y}_{2st}) = 2.913005764$. The 'Trace' is $1.869742767 + 2.913005764 = 4.782748531$.

6.3 Chatterjee's compromise allocation

Chatterjee (1967) obtained the compromise allocation by minimizing the sum of the relative increase E_l in the variances of the estimates $\bar{y}_{lst}$ of the population means $\bar{Y}_l$; $l = 1, 2, ..., p$.

Chatterjee formulated the problem as

$$\left.\begin{array}{l} \text{Minimize} \quad E = \sum_{l=1}^{p} E_l = \frac{1}{C_0} \sum_{l=1}^{p} \sum_{h=1}^{L} \frac{\hat{c}_h (n_{lh}^* - n_h)^2}{n_h} \\[4mm] \text{subject to} \quad \sum_{h=1}^{L} \hat{c}_h n_h + \sqrt{V(\hat{c}_h) n_h^2} \le C_0 \\[4mm] \text{and} \qquad 2 \le n_h \le N_h \; ; h = 1, 2, ..., L \end{array}\right\} \qquad (6.3)$$

where n_{lh}^* is the usual optimum allocation for fixed budget C_0 for the lth characteristic in h^{th} stratum.

With available data the Chatterjee's compromise allocation (rounded off) are obtained as: $n_1 = 12, n_2 = 13, n_3 = 09, n_4 = 17, n_5 = 17, n_6 = 15, n_7 = 24$. With variances (ignoring fpc) as $V_C(\bar{y}_{1st}) = 1.860335122$, $V_C(\bar{y}_{2st}) = 2.906798467$ and a trace value of 4.767133589, where 'C' stands for Chatterjee.

6.4 Sukhatme's compromise allocation

Sukhatme et al. (1984) obtained the compromise allocation by minimizing the sum of the variances for the p characteristics under linear cost constraints. The NLPP for this allocation is given as:

$$
\left.
\begin{aligned}
&\text{Minimize } V = \sum_{l=1}^{p} \sum_{h=1}^{L} \frac{W_h^2 s_{lh}^2}{n_h} \\
&\text{subject to } \sum_{l=1}^{p} \sum_{h=1}^{L} \hat{c}_h n_h + \sqrt{V(\hat{c}_h) n_h^2} \leq C_0 \\
&\text{and} \qquad 2 \leq n_h \leq N_h ; h = 1, 2, ..., L
\end{aligned}
\right\} \qquad (6.4)
$$

Using the values given in Table 1 and 2 the rounded off solution to NLPP (6.4) is given as

$n_1 = 14, n_2 = 15, n_3 = 11, n_4 = 18, n_5 = 19, n_6 = 17, n_7 = 27$ with the objective value as 4.240085924 which is also the trace value.

7. Summary of the Results

Table 3 gives the summary of the results of the numerical illustration.

Table 3 Summary of the results

The last column of Table 3 provides the Relative Efficiencies (R.E.) of the four compromise allocations as compared to the proportional allocation. It can be seen that the proposed allocation is the most efficient among the considered allocations.

Acknowledgement

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Table 1: Data for seven strata and two characteristics

h	N_h	W_h	s_{1h}	s_{2h}
1	472	0.1888	05.237	07.815
2	559	0.2236	05.821	07.949
3	425	0.1700	05.238	07.725
4	218	0.0872	25.528	30.125
5	233	0.0932	22.232	32.231
6	328	0.1312	15.129	18.455
7	265	0.1060	40.125	45.358

Table 2: Expected cost with their estimated variances for the two characteristics

h	$\hat{c}_{1h}$	$\hat{c}_{2h}$	$\hat{c}_h = \hat{c}_{1h} + \hat{c}_{2h}$	$\hat{\sigma}^2_{c_{1h}}$	$\hat{\sigma}^2_{c_{2h}}$	$\hat{\sigma}^2_{c_h} = \hat{\sigma}^2_{c_{1h}} + \hat{\sigma}^2_{c_{2h}}$
1	3	3	6	6	10	16
2	5	3	8	7	12	19
3	4	3	7	8	15	23
4	7	5	12	11	20	31
5	6	5	11	15	22	37
6	6	4	10	17	27	44
7	9	6	15	20	38	58

Table 3: Summary of the results

Allocation	Allocations							Trace	R.E. w.r.t. Proportional Allocation
	n_1	n_2	n_3	n_4	n_5	n_6	n_7		
(i) Proportional	23	28	21	11	11	16	13	6.501194863	1.0000
(ii) Cochran	12	13	10	16	17	15	24	4.782748531	1.359301
(iii) Chatterjee	12	13	09	17	17	15	24	4.767133589	1.363753447
(iv) Sukhatme	14	15	11	18	19	17	27	4.240085924	1.533269603
(v) Proposed	14	14	14	18	20	16	27	4.215147984	1.542340835

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