Stochastic Programming with Cauchy Distribution

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Abstract

The aim of this paper is to derive a method for solving a stochastic linear programming problem with Cauchy distribution. Assuming that the coefficients are distributed as Cauchy random variables, the stochastic linear programming is converted to a deterministic non-linear programming problem by a suitable transformation. Then an algorithm can be used to solve the resulting deterministic problem .A numerical example can be considered to illustrate the above methodology.

Keywords and phrases: Stochastic programming, Cauchy random variables, Probability density functions, Levy inversion theorem.

Introduction

Charnes and Cooper [6] first introduced the stochastic programming by taking different objective functions and constraints. Various models have been suggested by several researchers and most of the probabilistic model assumes normal distribution for model coefficients.

Here we consider a stochastic programming problem with Cauchy distribution. A stochastic linear programming problem can be presented as follows:

$$\text{Max } Z = \sum_{j=1}^{n} c_j x_j \ , \qquad (1)$$

Subject to the constraints,

$$\Pr ob\left(\sum_{j=1}^{n} a_{ij} x_j \le b_i \right) \ge 1 - \alpha_i \ , \ i = 1,2,..............., m \qquad (2)$$

$$x_j \ge 0, \quad j = 1,2,........., n,$$

Where $0 < \alpha_i < 1$ and are constants. It is assumed that a_{ij} are independent Cauchy random variables with known distributions for i =1,2,……………,m and j =1,2,……………,n. The main objective of this section is to find a solution for general stochastic programming problems involving Cauchy distribution in the constraints.

First we obtain the probability density function of the linear combination of 'n' independent Cauchy random variables, then using the probability density function, the probabilistic constraints are converted to the deterministic constraints, and then the resulting non-linear deterministic problem can be solved.

Probability distribution of $U_i = \sum_{i=1}^{n} a_{ij} x_j$ ***and the deterministic form of the probabilistic constraint***

Here we will find out the probability density function of random variable $U_i = \sum_{j=1}^{n} a_{ij} x_j$, where a_{ij}, j=1, 2……… n are independent Cauchy random variables and x_j, j=1, 2,……,n are variables .Then using it we find a deterministic form of the probabilistic constraints in a stochastic linear programming problem. The density function of U_i can be obtained by studying the special cases with n=2 and n=3. Let us consider a model involving only two random variables, the i^{th} probabilistic constraints can be stated by

$$\Pr ob(a_{i1}x_1 + a_{i2}x_2) \le b_i \ge 1 - \alpha_i \,,\ i = 1,2,..............., m$$

Where a_{i1} and a_{i2} are two independent Cauchy random variables

Let $U_i = a_{i1}x_1 + a_{i2}x_2$

Then the density function of U_i can be obtained as,

$$f(U) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{-itu}\varphi_u(t)dt \qquad \text{(Using Levy inversion theorem)}$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{-itu} e^{-(x_1+x_2)|t|} dt$$

$$= \frac{1}{2\pi}\left[\int_{-\infty}^{0} e^{-itu} e^{(x_1+x_2)t} dt + \int_{0}^{\infty} e^{-itu} e^{-(x_1+x_2)t} dt\right]$$

$$= \frac{1}{2\pi}\left[\int_{0}^{\infty} e^{itu} e^{-(x_1+x_2)t} dt + \int_{0}^{\infty} e^{-itu} e^{-(x_1+x_2)t} dt\right]$$

$$= \frac{1}{2\pi}\left[\int_{0}^{\infty} e^{itu} e^{-(x_1+x_2)t} dt + \int_{0}^{\infty} e^{-itu} e^{-(x_1+x_2)t} dt\right]$$

$$= \frac{1}{2\pi}\left[\int_{0}^{\infty} e^{-t(x_1+x_2-iu)} dt + \int_{0}^{\infty} e^{-t(x_1+x_2+iu)} dt\right]$$

$$= \frac{1}{2\pi}\left[\frac{e^{-t(x_1+x_2-iu)}}{-(x_1+x_2-iu)} + \frac{e^{-t(x_1+x_2+iu)}}{-(x_1+x_2+iu)}\right]_0^{\infty}$$

$$= \frac{1}{2\pi}\left[\frac{1}{(x_1+x_2-iu)} + \frac{1}{(x_1+x_2+iu)}\right]$$

$$= \frac{1}{2\pi}\left[\frac{2(x_1+x_2)}{\left((x_1+x_2)^2 - i^2u^2\right)}\right]$$

$$= \left[\frac{(x_1+x_2)}{\pi\left((x_1+x_2)^2 + u^2\right)}\right] \qquad (4)$$

Hence ith probabilistic constraints can be made deterministic constraints by integrating the probability density function of u_i as stated below,

$$\int_0^{b_i} f(u_i)du_i \ge 1-\alpha_i \quad i = 1,2,..............,m$$

This can be simplified as follows,

$$\int_0^{b_i} f(u_i)du_i = \frac{1}{\pi}\int_0^{b_i}\frac{(x_1+x_2)}{\left((x_1+x_2)^2 + u_i^2\right)}du_i \qquad (5)$$

If $u_i = a_{i1}x_1 + a_{i2}x_2$

Then $x_1 = \frac{u_i - a_{i2}x_2}{a_{i1}}$ and $x_2 = \frac{u_i - a_{i1}x_1}{a_{i2}}$

Now $\quad x_1 + x_2 = \frac{u_i - a_{i2}x_2}{a_{i1}} + \frac{u_i - a_{i1}x_1}{a_{i2}}$

$$= \frac{u_i(a_{i1}+a_{i2}) - \left(a_{i1}^2x_1 + a_{i2}^2x_2\right)}{a_{i1}a_{i2}}$$

Let $A = \frac{a_{i1}+a_{i2}}{a_{i1}a_{i2}}$ $\quad and\ B = \frac{a_{i1}^2x_1 + a_{i2}^2x_2}{a_{i1}a_{i2}}$

Then, $x_1 + x_2 = Au_i - B = p(Say)$

Now putting the above values in (5)

$$\int_0^{b_i} f(u_i)du_i = \frac{1}{\pi}\int_0^{b_i}\frac{(Au_i - B)}{\left((Au_i-B)^2 + u_i^2\right)}du_i$$

$$= \frac{1}{\pi}\int_{-\mathbf{B}}^{Ab_i-B}\frac{p}{A\left(p^2 + \left(\frac{p+B}{A}\right)^2\right)}dp$$

$$= \frac{1}{\pi}\int_{-\mathbf{B}}^{Ab_i-B}\frac{pdp}{A\left(p^2 + \left(\frac{p}{A}+\frac{B}{A}\right)^2\right)}$$

$$= \frac{1}{\pi}\int_{-\mathbf{B}}^{Ab_i-B}\frac{pdp}{A\left(p^2 + \left(\frac{p}{A}\right)^2 + \left(\frac{B}{A}\right)^2 + 2\left(\frac{p}{A}\right)\left(\frac{B}{A}\right)\right)}$$

$$=\frac{1}{\pi}\int_{-\mathbf{B}}^{Ab_i-B}\frac{pdp}{A\left\{\left(1+\frac{1}{A^2}\right)p^2+\left(\frac{2B}{A^2}\right)p+\frac{B^2}{A^2}\right\}}$$

$$=\frac{1}{\pi}\int_{-\mathbf{B}}^{Ab_i-B}\frac{pdp}{A\left\{\left(\frac{A^2+1}{A^2}\right)p^2+\left(\frac{2B}{A^2}\right)p+\frac{B^2}{A^2}\right\}}$$

$$=\frac{1}{\pi}\int_{-\mathbf{B}}^{Ab_i-B}\frac{pdp}{\left\{\left(\frac{A^2+1}{A}\right)p^2+\left(\frac{2B}{A}\right)p+\frac{B^2}{A}\right\}}$$

Let R=a+bt+ct^2, where $c=\frac{A^2+1}{A}$, $b=\frac{2B}{A}$ *and* $a=\frac{B^2}{A}$ then,

$\Delta=4ac-b^2=4B^2>0\,and\int\frac{tdt}{R}=\frac{1}{2c}\ln R-\frac{b}{2c}\int\frac{dt}{R}$ (c.f [42] , Page 68)

$$=\frac{1}{\pi}\left[\left\{\frac{1}{2\left(\frac{A^2+1}{A}\right)}\right\}\ln\left\{\left(\frac{A^2+1}{A}\right)p^2+\left(\frac{2B}{A}\right)p+\frac{B^2}{A}\right\}-\frac{\left(\frac{2B}{A}\right)}{2\left(\frac{A^2+1}{A}\right)}\int_{-\mathbf{B}}^{Ab_i-B}\frac{dp}{\left\{\left(\frac{A^2+1}{A}\right)p^2+\left(\frac{2B}{A}\right)p+\frac{B^2}{A}\right\}}\right]$$

$$=\frac{1}{\pi}\left[\left\{\frac{1}{2\left(\frac{A^2+1}{A}\right)}\right\}\ln\left\{\left(\frac{A^2+1}{A}\right)p^2+\left(\frac{2B}{A}\right)p+\frac{B^2}{A}\right\}-\frac{B}{A^2+1}\frac{2}{\sqrt{4B^2}}\,arctg\,\frac{\frac{2B}{A}+2\left(\frac{A^2+1}{A}\right)p}{\sqrt{4B^2}}\right]_{-B}^{Ab_i-B}$$

Because $\int\frac{dt}{R}=\frac{2}{\sqrt{\Delta}}arctg\left(\frac{b+2ct}{\sqrt{\Delta}}\right), if\Delta>0$ (c.f [42], Page 68)

$$=\frac{1}{\pi}\left[\left\{\frac{1}{2\left(\frac{A^2+1}{A}\right)}\right\}\ln\left\{\left(\frac{A^2+1}{A}\right)p^2+\left(\frac{2B}{A}\right)p+\frac{B^2}{A}\right\}-\frac{1}{A^2+1}arctg\left\{\frac{1}{A}+\left(\frac{A^2+1}{AB}\right)p\right\}\right]_{-B}^{Ab_i-B}$$

$$=\frac{1}{\pi}\left[\left\{\frac{1}{2\left(\frac{A^2+1}{A}\right)}\right\}\ln\left\{\left(\frac{A^2+1}{A}\right)(Ab_i-B)^2+\left(\frac{2B}{A}\right)(Ab_i-B)+\frac{B^2}{A}\right\}-\frac{1}{A^2+1}arctg\left\{\frac{1}{A}+\left(\frac{A^2+1}{AB}\right)(Ab_i-B)\right\}\right]$$

$$-\frac{1}{\pi}\left[\left\{\frac{1}{2\left(\frac{A^2+1}{A}\right)}\right\}\ln\left\{\left(\frac{A^2+1}{A}\right)(-B)^2+\left(\frac{2B}{A}\right)(-B)+\frac{B^2}{A}\right\}-\frac{1}{A^2+1}arctg\left\{\frac{1}{A}+\left(\frac{A^2+1}{AB}\right)(-B)\right\}\right]$$

$$=\frac{1}{\pi}\left[\left\{\frac{A}{2(A^2+1)}\right\}\ln\left\{\frac{(A^2+1)(A^2b_i{}^2+B^2-2Ab_iB)}{A}+\frac{2Ab_iB}{A}-\frac{2B^2}{A}+\frac{B^2}{A}\right\}-\frac{1}{A^2+1}arctg\left\{\frac{1}{A}+\frac{A^3b_i-A^2B+Ab_i-B}{AB}\right\}\right]$$

$$-\frac{1}{\pi}\left[\left\{\frac{A}{2(A^2+1)}\right\}\ln\left\{\frac{A^2B^2}{A}+\frac{B^2}{A}-\frac{2B^2}{A}+\frac{B^2}{A}\right\}-\frac{1}{A^2+1}arctg\left\{\frac{1}{A}-\frac{A^2B}{AB}-\frac{B}{AB}\right\}\right]$$

$$=\frac{1}{\pi}\left[\left\{\frac{A}{2(A^2+1)}\right\}\ln\left\{\frac{A^4b_i{}^2+A^2B^2-2A^3b_iB+A^2b_i{}^2+B^2-2Ab_iB}{A}+2b_iB-\frac{B^2}{A}\right\}-\frac{1}{A^2+1}arctg\left(\frac{1}{A}+\frac{A^2b_i}{B}+\frac{b_i}{B}-\frac{1}{A}-A\right)\right]$$

$$-\frac{1}{\pi}\left[\left\{\frac{A}{2(A^2+1)}\right\}\ln(AB^2)-\frac{1}{A^2+1}arctg(-A)\right]$$

$$=\frac{1}{\pi}\left[\frac{A}{2(A^2+1)}\left\{\ln\left(A^3b_i{}^2+AB^2-2A^2b_iB+Ab_i{}^2+\frac{B^2}{A}-2b_iB+2b_iB-\frac{B^2}{A}\right)-\ln(AB^2)\right\}-\frac{1}{A^2+1}\left\{arctg\left(\frac{A^2b_i}{B}+\frac{b_i}{B}-A\right)-arctg(-A)\right\}\right]$$

$$=\frac{1}{\pi}\left[\frac{A}{2(A^2+1)}\ln\left\{\frac{(A^2+1)Ab_i{}^2-2A^2b_iB+AB^2}{(AB^2)}\right\}-\frac{1}{A^2+1}\left\{arctg\left(\frac{A^2b_i}{B}+\frac{b_i}{B}-A\right)-arctg(-A)\right\}\right]$$

$$=\frac{1}{\pi}\left[\frac{A}{2(A^2+1)}\ln\left\{\frac{(A^2+1)Ab_i{}^2}{AB^2}-\frac{2A^2Bb_i}{AB^2}+\frac{(A^2+1)Ab_i{}^2}{AB^2}\right\}-\frac{1}{A^2+1}\left\{arctg\left(\frac{A^2b_i}{B}+\frac{b_i}{B}-A\right)-arctg(-A)\right\}\right]$$

$$= \frac{1}{\pi}\left[\frac{A}{2(A^2+1)} \ln\left\{ \left(\frac{A^2+1}{B^2} \right) b_i^{\,2} - \frac{2Ab_i}{B} + 1 \right\} - \frac{1}{A^2+1} \left\{ arctg\left(\frac{b_i}{B}(A^2+1) - A \right) - arctg(-A) \right\} \right]$$

On substituting the values of A and B,

$$\int_0^{b_i} f(u_i)du_i =$$

$$\frac{1}{\pi}\left[\begin{array}{l} \dfrac{\left(\dfrac{1}{a_{i1}}+\dfrac{1}{a_{i2}}\right)}{2\left\{\left(\dfrac{1}{a_{i1}}+\dfrac{1}{a_{i2}}\right)^2+1\right\}} \ln\left\{ \dfrac{\left(\dfrac{1}{a_{i1}}+\dfrac{1}{a_{i2}}\right)^2+1}{\left(x_1\dfrac{a_{i1}}{a_{i2}}+x_2\dfrac{a_{i2}}{a_{i1}}\right)^2} b_i^{\,2} - \dfrac{2\left(\dfrac{1}{a_{i1}}+\dfrac{1}{a_{i2}}\right)b_i}{\left(x_1\dfrac{a_{i1}}{a_{i2}}+x_2\dfrac{a_{i2}}{a_{i1}}\right)} + 1 \right\} \\ - \quad \dfrac{1}{\left(\dfrac{1}{a_{i1}}+\dfrac{1}{a_{i2}}\right)^2+1} \left\{ arctg\left(\dfrac{b_i\left(\left(\dfrac{1}{a_{i1}}+\dfrac{1}{a_{i2}}\right)^2+1\right)}{\left(x_1\dfrac{a_{i1}}{a_{i2}}+x_2\dfrac{a_{i2}}{a_{i1}}\right)} - \left(\dfrac{1}{a_{i1}}+\dfrac{1}{a_{i2}}\right) \right) - arctg - \left(\dfrac{1}{a_{i1}}+\dfrac{1}{a_{i2}}\right) \right\} \end{array} \right] \quad (6)$$

Proceeding as in above for the cases of three variables, the i[th] probabilistic constraints can be presented as,

$$\Pr ob(a_{i1}x_1 + a_{i2}x_2 + a_{i3}x_3) \le b_i \ge 1 - \alpha_i\,, \; i = 1,2,..............., m$$

Where a_{i1}, a_{i2} and a_{i3} are three independent Cauchy random variables

$$Let\ U_i = a_{i1}x_1 + a_{i2}x_2 + a_{i3}x_3$$

Then the density function of U_i can be obtained as,

$$f(U) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{-itu}\varphi_u(t)dt \qquad \text{(Using Levy inversion theorem)}$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{-itu} e^{-(x_1+x_2+x_3)|t|} dt$$

$$= \frac{1}{2\pi}\left[\int_{-\infty}^{0} e^{-itu} e^{(x_1+x_2+x_3)t} dt + \int_0^{\infty} e^{-itu} e^{-(x_1+x_2+x_3)t} dt \right]$$

$$= \frac{1}{2\pi}\left[\int_0^{\infty} e^{itu} e^{-(x_1+x_2+x_3)t} dt + \int_0^{\infty} e^{-itu} e^{-(x_1+x_2+x_3)t} dt \right]$$

$$= \frac{1}{2\pi}\left[\int_0^{\infty} e^{itu} e^{-(x_1+x_2+x_3)t} dt + \int_0^{\infty} e^{-itu} e^{-(x_1+x_2+x_3)t} dt \right]$$

$$= \frac{1}{2\pi}\left[\int_0^{\infty} e^{-t(x_1+x_2+x_3-iu)} dt + \int_0^{\infty} e^{-t(x_1+x_2+x_3+iu)} dt \right]$$

$$=\frac{1}{2\pi}\left[\frac{e^{-t(x_1+x_2+x_3-iu)}}{-(x_1+x_2+x_3-iu)}+\frac{e^{-t(x_1+x_2+x_3+iu)}}{-(x_1+x_2+x_3+iu)}\right]_0^\infty$$

$$=\frac{1}{2\pi}\left[\frac{1}{(x_1+x_2+x_3-iu)}+\frac{1}{(x_1+x_2+x_3+iu)}\right]$$

$$=\frac{1}{2\pi}\left[\frac{2(x_1+x_2+x_3)}{\left((x_1+x_2+x_3)^2-i^2u^2\right)}\right]$$

$$=\left[\frac{(x_1+x_2+x_3)}{\pi\left(x_1+x_2+{x_3}^2+u^2\right)}\right] \tag{7}$$

Hence ith probabilistic constraints can be made deterministic constraints by integrating the probability density function of u_i as stated below,

$$\int_0^{b_i} f(u_i)du_i \geq 1-\alpha_i \quad i=1,2,..............,m$$

This can be simplified as follows,

$$\int_0^{b_i} f(u_i)du_i=\frac{1}{\pi}\int_0^{b_i}\frac{(x_1+x_2+x_3)}{\left((x_1+x_2+x_3)^2+{u_i}^2\right)}du_i \tag{8}$$

If $u_i=a_{i1}x_1+a_{i2}x_2+a_{i3}x_3$

Then $x_1=\dfrac{u_i-a_{i2}x_2-a_{i3}x_3}{a_{i1}}$,

$x_2=\dfrac{u_i-a_{i1}x_1-a_{i3}x_3}{a_{i2}}$

and $x_3=\dfrac{u_i-a_{i1}x_1-a_{i2}x_2}{a_{i3}}$

Now $x_1+x_2+x_3=\dfrac{u_i-a_{i2}x_2-a_{i3}x_3}{a_{i1}}+\dfrac{u_i-a_{i1}x_1-a_{i3}x_3}{a_{i2}}+\dfrac{u_i-a_{i1}x_1-a_{i2}x_2}{a_{i3}}$

$$=u_i\left(\frac{1}{a_{i1}}+\frac{1}{a_{i2}}+\frac{1}{a_{i3}}\right)-\left\{x_1\left(\frac{a_{i1}}{a_{i2}}+\frac{a_{i1}}{a_{i3}}\right)+x_2\left(\frac{a_{i2}}{a_{i3}}+\frac{a_{i2}}{a_{i1}}\right)+x_1\left(\frac{a_{i3}}{a_{i1}}+\frac{a_{i3}}{a_{i2}}\right)\right\}$$

Let

$$A=\frac{a_{i1}+a_{i2}+a_{i3}}{a_{i1}a_{i2}a_{i3}}$$

$$and\ B=\left\{x_1\left(\frac{a_{i1}}{a_{i2}}+\frac{a_{i1}}{a_{i3}}\right)+x_2\left(\frac{a_{i2}}{a_{i3}}+\frac{a_{i2}}{a_{i1}}\right)+x_1\left(\frac{a_{i3}}{a_{i1}}+\frac{a_{i3}}{a_{i2}}\right)\right\}$$

Then, $x_1+x_2+x_3=Au_i-B=p(Say)$

Now putting the above values in (8)

$$\int_0^{b_i} f(u_i)du_i=\frac{1}{\pi}\int_0^{b_i}\frac{(Au_i-B)}{\left((Au_i-B)^2+{u_i}^2\right)}du_i$$

$$=\frac{1}{\pi}\left[\begin{array}{l}\dfrac{\left(\frac{1}{a_{i1}}+\frac{1}{a_{i2}}+\frac{1}{a_{i3}}\right)}{2\left\{\left(\frac{1}{a_{i1}}+\frac{1}{a_{i2}}+\frac{1}{a_{i3}}\right)^2+1\right\}}\ln\left\{\begin{array}{l}\dfrac{\left(\frac{1}{a_{i1}}+\frac{1}{a_{i2}}+\frac{1}{a_{i3}}\right)^2+1}{\left(x_1\left(\frac{a_{i1}}{a_{i2}}+\frac{a_{i1}}{a_{i3}}\right)+x_2\left(\frac{a_{i2}}{a_{i3}}+\frac{a_{i2}}{a_{i1}}\right)+x_1\left(\frac{a_{i3}}{a_{i1}}+\frac{a_{i3}}{a_{i2}}\right)\right)^2}b_i^{\,2}\\ -\dfrac{2\left(\frac{1}{a_{i1}}+\frac{1}{a_{i2}}+\frac{1}{a_{i3}}\right)b_i}{\left(x_1\left(\frac{a_{i1}}{a_{i2}}+\frac{a_{i1}}{a_{i3}}\right)+x_2\left(\frac{a_{i2}}{a_{i3}}+\frac{a_{i2}}{a_{i1}}\right)+x_1\left(\frac{a_{i3}}{a_{i1}}+\frac{a_{i3}}{a_{i2}}\right)\right)}+1\end{array}\right\}\\ -\dfrac{1}{\left(\frac{1}{a_{i1}}+\frac{1}{a_{i2}}+\frac{1}{a_{i3}}\right)^2+1}\left\{\begin{array}{l}arctg\left(\dfrac{b_i\left(\left(\frac{1}{a_{i1}}+\frac{1}{a_{i2}}+\frac{1}{a_{i3}}\right)^2+1\right)}{\left(x_1\left(\frac{a_{i1}}{a_{i2}}+\frac{a_{i1}}{a_{i3}}\right)+x_2\left(\frac{a_{i2}}{a_{i3}}+\frac{a_{i2}}{a_{i1}}\right)+x_1\left(\frac{a_{i3}}{a_{i1}}+\frac{a_{i3}}{a_{i2}}\right)\right)}-\left(\frac{1}{a_{i1}}+\frac{1}{a_{i2}}+\frac{1}{a_{i3}}\right)\right)\\ -arctg-\left(\frac{1}{a_{i1}}+\frac{1}{a_{i2}}+\frac{1}{a_{i3}}\right)\end{array}\right\}\end{array}\right] \quad (9)$$

Finally we generalize the results to 'n' independent random variables and the result may be stated as follows

$$\Pr ob\left(a_{i1}x_1+a_{i2}x_2+....................+a_{in}x_n\right)\le b_i\ge 1-\alpha_i\,,\; i=1,2,................,m$$

Where a_{i1} ,a_{i2} ,.................., a_{in} are 'n' independent Cauchy random variables

Let $U_i = a_{i1}x_1 + a_{i2}x_2 + \ldots\ldots\ldots\ldots\ldots + a_{in}x_n$

$$Then, \int_0^{b_i} f(u_i)du_i=\frac{1}{\pi}\int_0^{b_i}\frac{(Au_i-B)}{\left((Au_i-B)^2+u_i^{\,2}\right)}du_i$$

$$=\frac{1}{\pi}\left[\begin{array}{l}\dfrac{\left(\sum_{j=1}^{n}\frac{1}{a_{ij}}\right)}{2\left\{\left(\sum_{j=1}^{n}\frac{1}{a_{ij}}\right)^2+1\right\}}\ln\left\{\dfrac{\left(\sum_{j=1}^{n}\frac{1}{a_{ij}}\right)^2+1}{\left(\sum_{\substack{k=1\\k\ne j}}^{n}\sum_{j=1}^{n}x_j\frac{a_{ij}}{a_{ik}}\right)^2}b_i^{\,2}\;-\dfrac{2\left(\sum_{j=1}^{n}\frac{1}{a_{ij}}\right)b_i}{\left(\sum_{\substack{k=1\\k\ne j}}^{n}\sum_{j=1}^{n}x_j\frac{a_{ij}}{a_{ik}}\right)}+1\right\}\\ -\dfrac{1}{\left(\sum_{j=1}^{n}\frac{1}{a_{ij}}\right)^2+1}\left\{arctg\left(\dfrac{b_i\left(\left(\sum_{j=1}^{n}\frac{1}{a_{ij}}\right)^2+1\right)}{\left(\sum_{\substack{k=1\\k\ne j}}^{n}\sum_{j=1}^{n}x_j\frac{a_{ij}}{a_{ik}}\right)}-\left(\sum_{j=1}^{n}\frac{1}{a_{ij}}\right)\right)\;-arctg-\left(\sum_{j=1}^{n}\frac{1}{a_{ij}}\right)\right\}\end{array}\right] \quad (10)$$

Deterministic model of stochastic linear programming

Assuming a_{ij} to be independent Cauchy random variables, the stochastic linear programming model as stated in (1), (2) and (3) can be converted to a deterministic linear programming problem as

Maximize

$$E[Z]=\sum_{j=1}^{n}E[c_j]x_j \qquad (11)$$

Subject to the constraints,

$$=\frac{1}{\pi}\left[\begin{array}{l}\dfrac{\left(\sum_{j=1}^{n}\dfrac{1}{a_{ij}}\right)}{2\left\{\left(\sum_{j=1}^{n}\dfrac{1}{a_{ij}}\right)^2+1\right\}}\ln\left\{\dfrac{\left(\sum_{j=1}^{n}\dfrac{1}{a_{ij}}\right)^2+1}{\left(\sum_{\substack{k=1\\k\neq j}}^{n}\sum_{j=1}^{n}x_j\dfrac{a_{ij}}{a_{ik}}\right)^2}b_i^2-\dfrac{2\left(\sum_{j=1}^{n}\dfrac{1}{a_{ij}}\right)b_i}{\left(\sum_{\substack{k=1\\k\neq j}}^{n}\sum_{j=1}^{n}x_j\dfrac{a_{ij}}{a_{ik}}\right)}+1\right\}\\ -\dfrac{1}{\left(\sum_{j=1}^{n}\dfrac{1}{a_{ij}}\right)^2+1}\left\{arctg\left(\dfrac{b_i\left(\left(\sum_{j=1}^{n}\dfrac{1}{a_{ij}}\right)^2+1\right)}{\left(\sum_{\substack{k=1\\k\neq j}}^{n}\sum_{j=1}^{n}x_j\dfrac{a_{ij}}{a_{ik}}\right)}-\left(\sum_{j=1}^{n}\dfrac{1}{a_{ij}}\right)\right)-arctg-\left(\sum_{j=1}^{n}\dfrac{1}{a_{ij}}\right)\right\}\end{array}\right]\leq 1-\alpha_i \qquad (12)$$

$$x_j\geq 0 \qquad , j=1,2,......................,n \qquad (13)$$

Then we present a numerical example to illustrate the methodology.

Let, $a_{11}=5$, $a_{12}=4$, $a_{13}=8$, $a_{21}=10$, $a_{22}=2$, $a_{23}=20$

$b_1=10$, $b_2=20$, $\alpha_1=0.05$ *and* $\alpha_2=0.1$

Maximize, $Z=5x_1+6x_2+3x_3$

Subject to the constraints,

$$\frac{1}{\pi}\left[\begin{array}{l}\dfrac{0.575}{2.66125}\ln\left\{\dfrac{133.0625}{(1.875x_1+1.3x_2+3.6x_3)^2}-\dfrac{11.5}{(1.875x_1+1.3x_2+3.6x_3)}+1\right\}\\ \qquad -\left(\dfrac{1}{1.330625}\right)\left\{arctg\,\dfrac{133.0625}{(1.875x_1+1.3x_2+3.6x_3)}-(0.575)-arctg(-0.575)\right\}\end{array}\right]\leq 0.05$$

$$\frac{1}{\pi}\left[\begin{array}{l}\dfrac{0.65}{2.845}\ln\left\{\dfrac{56.9}{(5.5x_1+0.3x_2+12x_3)^2}-\dfrac{26}{(5.5x_1+0.3x_2+12x_3)}+1\right\}\\ \qquad -\left(\dfrac{1}{1.4225}\right)\left\{arctg\,\dfrac{28.45}{(5.5x_1+0.3x_2+12x_3)}-(0.65)-arctg(-0.65)\right\}\end{array}\right]\leq 0.01$$

$$x_1, x_2 \text{ and } x_3\geq 0$$

By using the Genetic Programming we found the optimum solution for the above program is

$$x_1 = 1.741, \quad x_2 = 1.748, \quad x_3 = 1.727 \quad and \quad E[Z] = 23.886$$

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