A note on "A New Approach for the Selection of Advanced Manufacturing Technologies: Data Envelopment Analysis with Double Frontiers"

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Recently, using the data envelopment analysis (DEA) with double frontiers approach, Wang and Chin (2009) proposed a new approach for the selection of advanced manufacturing technologies: DEA with double frontiers and a new measure for the selection of the best advanced manufacturing technologies (AMTs). In this note, we show that their proposed overall performance measure for the selection of the best AMT has an additional computational burden. Moreover, we propose a new measure for developing a complete ranking of AMTs. Numerical examples are examined using the proposed measure to show its simplicity and usefulness in the AMT selection and justification.

Keywords: Data envelopment analysis; Advanced manufacturing technology; Optimistic and pessimistic efficiencies.

1. Introduction

Selection of advanced manufacturing technologies (AMTs) is an important decision-making process for the explanation and implementation of AMTs. This requires careful consideration of various performance criteria (Wang & Chin, 2009). As an excellent method for performance evaluation based on data when a set of decision-making units (DMUs) has multiple inputs and outputs, data envelopment analysis (DEA) has proven its value. Therefore, the DEA has been widely used for AMT selection and justification.

For best use of the DEA, Wang and Chin (2009) introduced a new DEA method called "DEA with double frontiers" for AMTs selection and justification. The DEA with double frontiers considers two different efficiencies, i.e. optimistic and pessimistic efficiencies for decision-making. In this note, we show that the overall performance measure proposed by Wang and Chin (2009) for selecting the best AMT has an additional computational burden and may affect the ranking results. Finally, we propose a new measure to develop a complete ranking of AMTs.

The remainder of the paper is organized as follows: Section 2 starts with an overview on the measure proposed by Wang and Chin (2009). Then, it proposes a new overall performance measure for ranking AMTs. Numerical examples and conclusion are presented in sections 3 and 4, respectively.

2. DEA with double frontiers

2.1. Review on Wang and Chin's (2009) work

Assume that there are n AMTs for selection that must be evaluated in terms of m inputs and s outputs. For AMT_j ($j=1,\dots,n$), we show input values with x_{ij} ($i=1,\dots,m$) and output values with y_{rj} ($r=1,\dots,s$), all of which are known and non-negative. The optimistic efficiency of AMT_j compared to other AMTs is measured with the following CCR model (Charnes et al., 1978):

$$
\begin{aligned}
\max \quad & \theta_o = \sum_{r=1}^{s} u_r y_{ro} \\
\text{s.t.} \quad & \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \le 0, \quad j=1,\dots,n, \\
& \sum_{i=1}^{m} v_i x_{io} = 1, \\
& u_r, v_i \ge 0, \quad r=1,\dots,s; \quad i=1,\dots,m.
\end{aligned} \tag{1}
$$

where AMT_o is the AMT under evaluation, and u_r ($r=1,\dots,s$) and v_i ($i=1,\dots,m$) are decision variables. If there is a set of positive weights u_r^* ($r=1,\dots,s$) and v_i^* ($i=1,\dots,m$) to supply $\theta_o^*=1$, then AMT_o is called optimistic efficient; otherwise, it is called optimistic non-efficient.

In addition, the pessimistic efficiency of AMT_o compared to other AMTs can be measured with the following model (Azizi & Wang, 2013; Liu & Chen, 2009; Wang et al., 2007):

$$
\begin{aligned}
\min \quad & \varphi_o = \sum_{r=1}^{s} u_r y_{ro} \\
\text{s.t.} \quad & \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \ge 0, \quad j=1,\dots,n, \\
& \sum_{i=1}^{m} v_i x_{io} = 1, \\
& u_r, v_i \ge 0, \quad r=1,\dots,s; \quad i=1,\dots,m.
\end{aligned} \tag{2}
$$

When there is a set of positive weights u_r^* ($r=1,\dots,s$) and v_i^* ($i=1,\dots,m$) to supply $\varphi_o^*=1$, then AMT_o is called pessimistic inefficient; otherwise, it is called pessimistic non-inefficient.

Optimistic and pessimistic efficiencies are measured from different perspectives, and often lead to two different rankings for AMTs. Therefore, an overall performance measure is needed to obtain a single overall ranking of AMTs. To this end, Wang and Chin (2009) proposed the following overall performance measure for ranking AMTs:

$$
\eta_j = \frac{\theta_j^*}{\sqrt{\sum_{i=1}^{n} \theta_i^{*2}}} + \frac{\varphi_j^*}{\sqrt{\sum_{i=1}^{n} \varphi_i^{*2}}}, \quad j=1,\dots,n \tag{3}
$$

where θ_j^* and φ_j^* are the optimistic and pessimistic efficiencies of AMT_j, respectively.

Measure (3) has an additional computational burden, because if we assume the vectors $\vec{\theta} = (\theta_1^*, \ldots, \theta_n^*)$ and $\vec{\varphi} = (\varphi_1^*, \ldots, \varphi_n^*)$ are the vectors for optimistic and pessimistic efficiencies, respectively and the vectors $\vec{\overline{\theta}} = (\overline{\theta}_1^*, \ldots, \overline{\theta}_n^*)$ and $\vec{\overline{\varphi}} = (\overline{\varphi}_1^*, \ldots, \overline{\varphi}_n^*)$ are the normalized vectors for optimistic and pessimistic efficiencies based on the Euclidean norm, respectively, then we have:

$$\begin{aligned} \overline{\theta}_j^* &= \frac{\theta_j^*}{\sqrt{\sum_{i=1}^n \theta_i^{*2}}}, \quad j = 1, \ldots, n, \\ \overline{\varphi}_j^* &= \frac{\varphi_j^*}{\sqrt{\sum_{i=1}^n \varphi_i^{*2}}}, \quad j = 1, \ldots, n \end{aligned} \tag{4}$$

It is clear that the overall performance measure defined in (3) is the sum of elements for the normalized vectors of the two vectors derived from optimistic and pessimistic efficiencies. Since the normalization of efficiency vectors has no effect on the ranking of AMTs, the following measure can also be used for ranking AMTs:

$$x_j = \theta_j^* + \varphi_j^*, \quad j = 1, \ldots, n \tag{5}$$

Measure (5) may provide more correct results compared with measure (3), because measure (3) includes a rounding error.

2.2. New overall performance measure

In Wang et al. (2007), the geometric average of two efficiencies was proposed as the overall performance measure. The geometric average efficiency integrates both optimistic and pessimistic efficiency measures for each DMU, so it is more comprehensive than either of these two measures. In Wang and Chin (2009), in a sense, the arithmetic average of both optimistic and pessimistic efficiencies was proposed as an overall performance measure. Since measure (3) is twice the arithmetic average of the normalized efficiencies and their ranking is exactly the same, three different means (i.e., geometric average, arithmetic average, and quadratic mean) can be used for ranking DMUs as follows:

$$G_j = \sqrt{\theta_j^* \cdot \varphi_j^*}, \quad j = 1, \ldots, n \tag{6}$$

$$A_j = \frac{\theta_j^* + \varphi_j^*}{2}, \quad j = 1, \ldots, n \tag{7}$$

$$Q_j = \sqrt{\frac{\theta_j^{*2} + \varphi_j^{*2}}{2}}, \quad j = 1, \ldots, n \tag{8}$$

The relationship between these means is as follows:

$$G_j \le A_j \le Q_j, \quad j = 1, \ldots, n \tag{9}$$

Generally, when optimistic and pessimistic efficiencies are larger, the DMU is evaluated better. Thus, according to equation (9), one can use the quadratic mean as the overall performance measure for ranking DMUs. Since the value $1/\sqrt{2}$ does not affect the ranking of DMUs, we consider the following measure as the new overall performance measure for each DMU:

$$Q_j = \sqrt{\theta_j^{*2} + \varphi_j^{*2}}, \quad j = 1, \ldots, n \tag{10}$$

3. Numerical Examples

In this section, we examine four numerical examples presented in Wang and Chin (2009) with measure (10). Comparison with the results of Wang and Chin (2009) is also presented wherever possible.

For input and output data related to all the tables presented in Wang and Chin (2009), we run DEA models (1) and (2) for each AMT to obtain optimistic and pessimistic efficiencies. The results are shown in Tables 1-4. Additionally, the overall performance of each AMT is measured by measures (3) and (10) and their ranking is shown in Tables 1-4.

Table 1: Evaluation of the 12 FMSs by DEA with double frontiers

FMS	Optimistic efficiency	Pessimistic efficiency	Measure (3)	Ranking based on measure (3)	Measure (10)	Ranking based on measure (10)
1	1.0000	1.0146	0.5670	7	1.4246	7
2	1.0000	1.0000	0.5631	8	1.4142	8
3	0.9824	1.1193	0.5898	5	1.4892	5
4	1.0000	1.1921	0.6144	2	1.5560	2
5	1.0000	1.2227	0.6226	1	1.5796	1
6	1.0000	1.1515	0.6036	4	1.5251	4
7	1.0000	1.1587	0.6055	3	1.5306	3
8	0.9614	1.0748	0.5717	6	1.4421	6
9	1.0000	1.0000	0.5631	8	1.4142	8
10	0.9536	1.0000	0.5494	11	1.3818	11
11	0.9831	1.0000	0.5581	10	1.4023	10
12	0.8012	1.0000	0.5043	12	1.2814	12

The AMTs ranking results based on the values obtained from measures (3) and (10), reported in Tables 1 and 2, show that the ranks are identical. But the ranking results obtained in Tables 3 and 4 are not identical. In Table 3, the ranking of AMTs 5, 8, 10, 11, 13, 15, 17, 18, 20, and 21 obtained according to measures (3) and (10) is not the same. Consider, for example AMTs 8 and 10. If we rank them by measure (5), ($x_{10} = 2.0803$ and $x_8 = 2.0715$), their ranking is switched. One of its reasons is the high computational burden of measure (3), and a rounding error. It is clear that measure (10) is more efficient, and can save a lot of calculations compared with measure (3). A similar problem exists in Table 4. The ranking based on measures (3) and (10) has changed the results of 26 AMTs. That is, more than 55% of AMTs are ranked wrongly. We have shown them in bold font. This is the biggest advantage of measure (10) over measure (3) for AMT selection and justification.

Table 2: Evaluation of the 12 industrial robots by DEA with double frontiers

Robot	Optimistic efficiency	Pessimistic efficiency	Measure (3)	Ranking based on measure (3)	Measure (10)	Ranking based on measure (10)
1	1.0000	1.0146	0.5670	7	1.4246	7
2	1.0000	1.0000	0.5631	8	1.4142	8
3	0.9824	1.1193	0.5898	5	1.4892	5
4	1.0000	1.1921	0.6144	2	1.5560	2
5	1.0000	1.2227	0.6226	1	1.5796	1
6	1.0000	1.1515	0.6036	4	1.5251	4
7	1.0000	1.1587	0.6055	3	1.5306	3
8	0.9614	1.0748	0.5717	6	1.4421	6
9	1.0000	1.0000	0.5631	8	1.4142	8
10	0.9536	1.0000	0.5494	11	1.3818	11
11	0.9831	1.0000	0.5581	10	1.4023	10
12	0.8012	1.0000	0.5043	12	1.2814	12

Table 3: Evaluation 21 the CNC lathes by DEA with double frontiers

CNC lathe	Optimistic efficiency	Pessimistic efficiency	Measure (3)	Ranking based on measure (3)	Measure (10)	Ranking based on measure (10)
1	1.0000	1.2133	0.4561	6	1.5723	6
2	0.8351	1.1183	0.3997	18	1.3957	18
3	0.8746	1.3936	0.4583	5	1.6453	5
4	1.0000	1.8121	0.5630	1	2.0697	1
5	0.9345	1.0833	0.4172	14	1.4307	**15**
6	0.8177	1.0000	0.3744	20	1.2917	20
7	0.5401	1.0000	0.3079	21	1.1365	21
8	1.0000	1.0715	0.4308	12	1.4657	**13**
9	1.0000	1.1634	0.4472	7	1.5341	7
10	0.8457	1.2346	0.4230	13	1.4965	**11**
11	0.8193	1.1960	0.4097	16	1.4497	**14**
12	1.0000	1.3867	0.4871	3	1.7096	3
13	0.8889	1.2326	0.4329	10	1.5197	**8**
14	1.0000	1.3929	0.4882	2	1.7147	2
15	1.0000	1.0785	0.4321	11	1.4708	**12**
16	0.9625	1.1476	0.4354	9	1.4978	9
17	0.9182	1.0691	0.4108	15	1.4092	**16**
18	0.8983	1.0581	0.4040	17	1.3880	**19**
19	0.9144	1.4144	0.4715	4	1.6842	4
20	0.7576	1.1879	0.3935	19	1.4089	**17**
21	0.9835	1.1285	0.4370	8	1.4969	**10**

Table 4: Evaluation of the 47 alternative machine component grouping solutions by DEA with double frontiers

Layout (DMU)	Optimistic efficiency	Pessimistic efficiency	Measure (3)	Ranking based on measure (3)	Measure (10)	Ranking based on measure (10)
1	1.0000	1.6410	0.3312	8	1.9217	**12**
2	0.9765	1.6350	0.3266	11	1.9044	**13**
3	0.9697	1.6184	0.3238	14	1.8867	14
4	0.9521	1.5419	0.3133	19	1.8122	19
5	0.7887	1.4328	0.2750	29	1.6355	**28**
6	0.9591	1.6670	0.3269	9	1.9232	**10**
7	0.9417	1.5932	0.3166	15	1.8507	**17**
8	0.8656	1.3382	0.2785	27	1.5937	**30**
9	1.0000	1.0000	0.2674	32	1.4142	**38**
10	1.0000	1.9342	0.3603	2	2.1774	2
11	0.9224	1.4970	0.3038	21	1.7584	21
12	0.9450	1.7519	0.3330	7	1.9905	7
13	1.0000	1.9648	0.3634	1	2.2047	1
14	0.9939	1.8523	0.3512	4	2.1021	4
15	0.9715	1.7503	0.3373	6	2.0019	6
16	0.7961	1.1808	0.2512	37	1.4241	37
17	0.8159	1.2558	0.2620	34	1.4976	34
18	0.9501	1.5561	0.3143	18	1.8232	18
19	0.9549	1.6723	0.3267	10	1.9257	**9**
20	0.9972	1.8763	0.3541	3	2.1249	3
21	0.9606	1.7879	0.3392	5	2.0296	5
22	0.9471	1.6722	0.3254	12	1.9218	**11**
23	0.9264	1.6030	0.3150	17	1.8514	**16**
24	0.7611	1.1179	0.2390	39	1.3523	**40**
25	0.6102	1.0000	0.2020	46	1.1715	46
26	0.8670	1.3969	0.2846	26	1.6441	26
27	0.8442	1.4961	0.2906	24	1.7178	**23**
28	0.9316	1.6973	0.3253	13	1.9362	**8**
29	0.9176	1.6272	0.3160	16	1.8680	**15**
30	0.9006	1.5412	0.3046	20	1.7851	20
31	0.8829	1.4675	0.2943	22	1.7126	**24**
32	0.7346	1.0554	0.2284	42	1.2859	**41**
33	0.5839	1.0000	0.1975	47	1.1580	47
34	0.7453	1.2478	0.2493	38	1.4534	**36**
35	0.7229	1.3544	0.2561	36	1.5352	**33**
36	0.7755	1.4448	0.2740	30	1.6398	**27**
37	0.8761	1.4779	0.2941	23	1.7181	**22**
38	0.8607	1.4144	0.2853	25	1.6557	25
39	0.8417	1.3581	0.2765	28	1.5978	**29**
40	0.7072	1.0000	0.2183	45	1.2248	45
41	0.7003	1.0565	0.2227	43	1.2675	**44**
42	0.6826	1.0836	0.2224	44	1.2807	**42**
43	0.6717	1.1789	0.2301	41	1.3568	**39**
44	0.8115	1.3569	0.2713	31	1.5810	31
45	0.8039	1.3097	0.2653	33	1.5367	**32**
46	0.8032	1.2585	0.2601	35	1.4930	35
47	0.7969	1.0000	0.2333	40	1.2787	**43**

4. Conclusion

In this note, we point to computational errors in the paper by Wang and Chin (2009). We showed that their proposed measure for ranking AMTs can be problematic. To overcome these problems, we proposed another measure for ranking AMTs. Numerical examples show that the proposed measure can rank all AMTs correctly. The proposed measure is expected to play an important role in AMT selection and justification and to have more applications in the future.

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