# Fuzzy Multi-objective Linear Programming Approach for Traveling Salesman Problem

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## Abstract

Traveling salesman problem (TSP) is one of the challenging real-life problems, attracting researchers of many fields including Artificial Intelligence, Operations Research, and Algorithm Design and Analysis. The problem has been well studied till now under different headings and has been solved with different approaches including genetic algorithms and linear programming. Conventional linear programming is designed to deal with crisp parameters, but information about real life systems is often available in the form of vague descriptions. Fuzzy methods are designed to handle vague terms, and are most suited to finding optimal solutions to problems with vague parameters. Fuzzy multi-objective linear programming, an amalgamation of fuzzy logic and multi-objective linear programming, deals with flexible aspiration levels or goals and fuzzy constraints with acceptable deviations.

In this paper, a methodology, for solving a TSP with imprecise parameters, is deployed using fuzzy multi-objective linear programming. An example of TSP with multiple objectives and vague parameters is discussed.

## 1. Introduction

The Traveling Salesman Problem is well-known among NP-hard combinatorial optimization problems. It represents a class of problems which are analogous to finding the least-cost sequence for visiting a set of cities, starting and ending at the same city in such a way that each city is visited exactly once. The desire of economy, in which least time span or least distance are also significant for a decision maker, ultimately poses TSP as a multi-objective problem.

In TSP as a Multi-Objective Combinatorial Optimization Problem, each objective function is represented in a distinct dimension. Of this form, to decide the multi objective TSP in the optimality means to determine the k-dimensional points that pertaining to the space of feasible solutions of the problem and that possess the minimum possible values according to all dimension. The permissible deviation from a specified value of a structural dimension is also considerable because traveling sales man can face a situation in which he is not able to achieve his objectives completely. There must be a set of alternatives from which he can select one that best meets his aspiration level. Conventional programming approaches does not deal with this situation however some researches have specifically treated the multi-objective TSP. Fischer and Richter (1982) used a branch and bound approach to solve a TSP with two (sum) criteria. Gupta and Warburton (1986) used the 2- and 3-opt heuristics for the maxordering TSP. Sigal (1994) proposed a decomposition approach for solving the TSP with respect to the two criteria of the route length and bottlenecking, where both objectives are obtained from the same cost matrix. Tung (1994) used a branch and bound method with a multiple labeling scheme to keep track of possible Paretooptimal tours. Melamed and Sigal (1997) suggested an *e*-constrained-based algorithm for bi-objective TSP. Ehrgott (2000) proposed an approximation algorithm with worst case performance bound. Hansen (2000) applied the tabu search algorithm to multi objective TSP. Borges and Hansen (2002) used the weighted sums program to study the global convexity for multi-objective TSP. Jaszkiewicz (2002) proposed the genetic local search which combines ideas from evolutionary algorithms, local search with modifications of the aggregation of the objective functions. Paquete and Stützle (2003) proposed the two-phase local search procedure to tackle bi-objective TSP. During the first phase, a good solution to one single objective is found by using an effective single objective algorithm. This solution provides the starting point for the second phase, in which a local search algorithm is applied to a sequence of different aggregations of the objectives, where each aggregation converts the bi-objective problem into a single objective one. Yan et al (2003) used an evolutionary algorithm to solve multi objective TSP. Angel, Bampis and Gourvès (2004) proposed the dynasearch algorithm which uses local search with an exponential sized neighborhood that can be searched in polynomial time using dynamic programming and a rounding technique. Paquete, Chiarandini and Stützle (2004) suggested a Pareto local search method which extends local search algorithm for the single objective TSP to bi-objective case. This method uses an archive to hold non-dominated solutions found in the search process.

Furthermore, in TSP the salesman takes decision of selecting an optimal and feasible route between any couple of cities on the basis of expected measures. In most of the real world problems it is not possible to have all constraints and resources in exact form rather they are in expected or vague form. This leads to the concept of fuzzy logic which enables us to emulate the human reasoning process and make decisions based on vague or imprecise data, and fuzzy programming gives the methodology of solving the problems in fuzzy environment.

An ideal solution method would solve every TSP problem to optimality, but this is not practical in most large problems. While advances have been made in solving the TSP, those advances have come at the cost of more complicated computer code. The complexity involves not only the length of the code, but the required nesting and data structures.

It is required to meet the aspiration level of a decision maker under which the current optimal solution remains still optimal and feasible. In this paper we propose a methodology which deals with vague parameters and achieve certain aspiration level of optimality for multi-objective TSP by transforming it into a linear program using fuzzy multi-objective linear programming.

This paper is structured as follows: Fuzzy programming in section 2, Fuzzy Multi-objective Linear Programming for TSP in section 3 and finally in section 4 a case study for TSP problem is given and solved with fuzzy multi-objective linear programming. Conclusion and future work are discussed in section 5 and section 6 respectively.

## 2. Fuzzy Programming

The methodology Fuzzy Multi-objective Linear programming, utilized here to derive algorithm of TSP is based on linear programming, multi-objective linear programming and fuzzy logic. The succinct synopsis of these is discussed in the proceeding sections.

### 2.1 Fuzzy Logic and Fuzzy Membership Functions

Fuzzy logic, introduced by Zadeh (1965), is a superset of conventional (Boolean) logic, which has been extended to handle the concept of partial truth: truth values between "completely true" and "completely false". As its name suggests, it is the logic underlying modes of reasoning which are approximate rather than exact. . Linguistic terms can better represent experience and subjective viewpoint of decision makers in more intuitive way and natural language format. The importance of fuzzy logic derives from the fact that most modes of human reasoning and especially common sense reasoning are approximate in nature. Fuzzy set theory uses linguistic variables rather than quantitative variables to represent imprecise concepts. A membership function of a fuzzy set, called fuzzy membership function, is mapped on interval [0, 1] which is an arbitrary grade of truth. The notation for the fuzzy membership function $\square_{\square}$ *(I)* of a set *A* is $\square_{\square}$: X → [0, 1].

### 2.2 Linear Programming

The first formal representation of a linear programming (LP) problem and an efficient technique for solving it was developed by George B. Dantzig (1947). The general linear programming model, for maximization problem, proposed by Dantzig is

$$\max z = \sum_{i=1}^{n} c_i \, x_i$$

Subject to

$$\sum_{i=1}^{n} a_{ij} \, x_{ij} \le b_i \qquad (j = 1,2,...m)$$

and $x_i >= 0$

Where $z$ is the objective function, $x_i$ are the decision variables, $m$ is the number of constraints, $n$ is the number of decision variables, and $b_i$ are the given resources. This linear programming model can be solved by different methods e.g. graphical solution, simplex method, etc.

## 2.3 Multi-objective Linear Programming

The limitations of linear programming are that it can deal only with single objective function and it does not incorporate the soft constraints. Zeleny in 1974 introduced the concept of multi-objective linear programming. A general linear multiple criteria decision making model can be presented as: find a vector x written in following form

$$x^T = [x_1, x_2, ...., x_n]$$

which maximize $k$ objective functions, with $n$ variables, and $m$ constraints is as follows

$$\max z_i = \sum_{j=1}^{n} c_{ij} x_j \quad (i = 1,2,...k)$$

Subject to

$$\sum_{i=1}^{n} a_{ij} x_i \le b_j \quad (j = 1,2,...m)$$

where the parameters $c_{ij}$ , $a_{ij}$ , and $b_j$ are given crisp values. In a precise form, the multiple objective problems can be represented by the following multi-objective linear programming model

$$\text{opt} \quad Z = CX$$
$$\text{s.t. } AX \le b$$

Where $Z = (z_1, z_2, ..., z_n)$ is the vector of objectives, $C$ is a $K$ x $N$ matrix of constants and $X$ is an $N$ x 1 vector of decision variables, $A$ is an $M$ x $N$ matrix of constants and $b$ is $M$ x 1 vector of constants.

## 2.4 Fuzzy Multi-objective Linear Programming

R. Bellman and L.A. Zadeh first proposed the concept of decision making in a fuzzy environment involving several objectives and H.J. Zimmerman (1978) applied their approaches to a vector-maximum problem. He transformed the fuzzy multi-objective linear programming problem to a classic single objective linear program.

Consider the following multi-objective linear programming model

$$\text{Max} \quad Z = CX$$

Subject to

$$AX \lesssim b$$

An adopted fuzzy model due to Zimmerman is

Max $CX \underline{>} \sim Z^0$

Subject to

$AX \underline{<} \sim b$

Where $Z^0 = (z_1^0, z_2^0 \ldots z_n^0)$ are the goals or aspiration levels and $>\sim$ and $\underline{<}\sim$ are the fuzzy inequalities that are the fuzzifications of ≥ and ≤ respectively. For measurement of satisfaction levels of objectives and constraints Zimmerman suggested the simplest kind of membership function

$$\mu_{1k}(C_k X) = \begin{cases} 0 & \text{if } C_k X \le z_k^0 - t_k \\ 1 - (z_k^0 - C_k X)/t_k & \text{if } z_k^0 - t_k \le C_k X \le z_k^0 \\ 1 & \text{if } C_k X \ge z_k^0 \end{cases}$$

Where k=1,…,n and $t_k$ is the admissible violation for the objective $z_k$., which is decided by the decision maker. Zimmerman discussed the membership function for maximizing objective function. In case of minimization objective function, the fuzzy membership function will be as follows

$$\mu_{1k}(C_k X) = \begin{cases} 0 & \text{if } C_k X \ge z_k^0 + t_k \\ 1 - (C_k X - z_k^0)/t_k & \text{if } z_k^0 \le C_k X \le z_k^0 + t_k \\ 1 & \text{if } C_k X \le z_k^0 \end{cases}$$

Where k=1,…,n.

Another class of fuzzy membership functions has $\mu_{2i}(a_i X)$ for ith constraint suggested by Zimmerman

$$\mu_{2i}(a_i X) = \begin{cases} 0 & \text{if } a_i X \ge b_i + d_i \\ 1 - (a_i X - b_i)/d_i & \text{if } b_i \le a_i X \le b_i + d_i \\ 1 & \text{if } a_i X \le b_i \end{cases}$$

where i =1, 2, …, M and $d_i$ is the admissible violation for fuzzy resource $b_i$ for ith constraint. These membership functions express the satisfaction of the decision maker with the solution so they must be maximized. As a result the objective function becomes

$$\max_X \left(\mu_{11}(C_1X), \mu_{12}(C_2X), \ldots, \mu_{1k}(C_kX), \mu_{21}(a_1X), \mu_{22}(a_2X), \ldots, \mu_{2m}(a_mX)\right)$$

According to the fuzzy set theorem, the membership function of the intersection of any two or more sets is the minimum membership function of these sets. By applying this theorem the objective becomes:

$$\max_X \min\left(\mu_{11}(C_1X), \mu_{12}(C_2X), \ldots, \mu_{1k}(C_kX), \mu_{21}(a_1X), \mu_{22}(a_2X), \ldots, \mu_{2m}(a_mX)\right)$$

From above representation, the fuzzy program can be rewritten as:

Max $\alpha$

Subject to

$\alpha \le 1-(z_k^0 - C_k X)/t_k$ k =1,…,n

$\alpha \le 1-(a_i X - b_i)/d_i$ i = 1,…, M $\alpha \ge 0$ X>=0 □ ∈ □ı□

Where $\alpha$ is overall satisfaction level achieved with respect to solution.

## 3. Fuzzy Multi-Objective Linear Programming Approach for TSP

The most frequently considered objective of the TSP is to determine an optimal order for traveling all the cities so that the total cost is minimized. Consider the situation when decision maker has to determine the optimal solution of TSP with minimized cost, time and overall distance. The individual objective functions can be formed for all the objectives of decision maker.

Let $x_{ij}$ represents the link from city *i* to city *j* and

$$X_{ij} = \begin{cases} 1 & \text{if city j is visited from city i} \\ 0 & \text{otherwise} \end{cases}$$

Let $c_{ij}$ be the cost of traveling from city *i* to city *j*, the overall cost of a particular route is the sum of the costs on the links comprising the route. Since the decision maker has to minimize the overall cost of traveling, so he can set goal for the total estimated cost of the entire route for the TSP denoted by $z_1^0$. But there can be the situations when the estimated cost doesn't meet and so the decision maker can set tolerance for the estimated cost. Let us denote the tolerance against this goal as $t_1$, the objective function for minimization of cost is given as:

$$z_1 \ : \min \sum_{i}^{n} \sum_{j}^{n} c_{ij}\, x_{ij} \le\sim z_1^0$$

Now let $d_{ij}$ be the distance from city *i* to city *j*. Let $z_2^0$ be the aspiration level for the objective function for minimization of distance, and $t_2$ be the tolerance, then the objective function takes the form: $z_2 \ : \min \sum_{i}^{n} \sum_{j}^{n} d_{ij}\, x_{ij} \le\sim z_2^0$

Now let $t_{ij}$ be the time spent in traveling from city i to city j, $z_3^0$ be the aspiration level for the objective function for minimization of total time, and $t_3$ be the corresponding tolerance. The objective function can be written as:

$$z_3 \ : \min \sum_{i}^{n} \sum_{j}^{n} t_{ij}\, x_{ij} \le\sim z_3^0$$

One important aspect is dependency of objective functions on each other. Most of the times they are dependent, but determining exact form of dependency is also a complex process. The proposed framework works in all the cases, if there exist some feasible solution.

These multiple objective can be represented in vector form of section-2, comprising multiple objectives with specified goals and tolerances. The membership functions can be set for these individual objective functions to check their level of acceptability.

We have the restriction in TSP that every city should be visited from exactly one its neighboring city, and vice versa. i.e.

$$\sum_{i}^{n} x_{ij} = 1 \quad \text{for all } j$$

$$\sum_{i}^{n} x_{ij} = 1 \quad \text{for all } i$$

A route can not be selected more than once. i.e.

$$x_{ij} + x_{ji} \leq 1 \qquad \text{for all } i, j$$

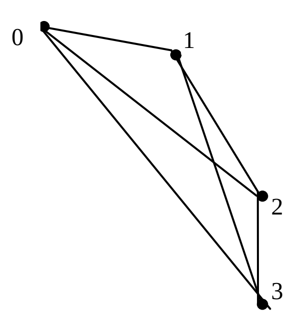
And the non-negativity constraints;

$$x_{ij} \geq 0.$$

Now these constraints can collectively be expressed in the vector form and the fuzzy membership functions can be defined for all the objective functions. Finally a linear model can be formulated using fuzzy multi-objective linear model using TSP objective functions, constraints and their corresponding membership functions. The model can be solved by mixed integer linear programming. The table-1 gives the matrix for time, cost and distance for each couple of cities.

## 4. Case Study for TSP

A traveling salesman has been analyzed with symmetric TSP, who starts from his home city 0; has to visit the three cities exactly once and he is required to comeback to his home city 0 by adopting a route with minimum cost, time and distance covered. A map of the cities to be visited is shown in Fig-1 and the cities listed along with their cost, time and distance matrix in table-1, where triple (c,d,t) represents; cost in rupees, distance in kilometers, and time in hours respectively for the corresponding couple of cites.




(Fig-1)

**Table 1:**

| City | 0<br>c ,d ,t | 1<br>c ,d ,t | 2<br>c ,d ,t | 3<br>c ,d ,t |
|---|---|---|---|---|
| **0** | 0, 0, 0 | 20,5,4 | 15,5,5 | 11,3,2 |
| **1** | 20,5,4 | 0, 0, 0 | 30,5,3 | 10,3,3 |
| **2** | 15,5,5 | 30,5,3 | 0, 0, 0 | 20,10,2 |
| **3** | 11,3,2 | 10,3,3 | 20,10,2 | 0, 0, 0 |

Let links $X_{ij}$ be the decision variable of selection of link (i, j) from city i to city j. Components of the linear program for the given problem are

**Decision variables:**

$$X_{ij} = \begin{cases} 1 & \text{if city } j \text{ is visited from city } i \\ 0 & \text{otherwise} \end{cases}$$

The three objective function $z_1, z_2, z_3$ are formulated for cost, distance and time respectively. Their Aspiration levels are set as 65, 16, 11 by solving each objective function subject to the given constraints in the TSP and their corresponding tolerances are decided as 5, 2, 1.

**Objective functions:**

$$\min\ z_1 = 20X_{01} + 15X_{02} + 11X_{03} + 20X_{10} + 30X_{12} + 10X_{13} + 15X_{20} + 30X_{21} + 20X_{23} + 11X_{30} + 10X_{31} + 20X_{32} \lesssim 65 \qquad (4.1)$$

Tolerance = $t_1 = 5$

$$\min\ z_2 = 5X_{01} + 5X_{02} + 3X_{03} + 5X_{10} + 5X_{12} + 3X_{13} + 5X_{20} + 5X_{21} + 10X_{23} + 3X_{30} + 3X_{31} + 10X_{32} \lesssim 16 \qquad (4.2)$$

Tolerance = $t_2 = 2$

$$\min\ z_3 = 4X_{01} + 5X_{02} + 2X_{03} + 4X_{10} + 3X_{12} + 3X_{13} + 5X_{20} + 3X_{21} + 2X_{23} + 2X_{30} + 3X_{31} + 2X_{32} \lesssim 11 \qquad (4.3)$$

Tolerance = $t_3 = 1$

The fuzzy membership function for cost, distance and time objective function are given as under based on equation (4.1), (4.2) and (4.3).

$$\mu(z_1) = \begin{cases} 0 & \text{if } z_1 \geq 70 \\ 1 - (z_1 - 65)/5 & \text{if } 65 \leq z_1 \leq 70 \\ 1 & \text{if } z_1 \leq 65 \end{cases}$$

$$\mu(z_2) = \begin{cases} 0 & \text{if } z_2 \geq 18 \\ 1-(z_2-16)/2 & \text{if } 16 \leq z_2 \leq 18 \\ 1 & \text{if } z_1 \leq 16 \end{cases}$$

$$\mu(z_3) = \begin{cases} 0 & \text{if } z_3 \geq 12 \\ 1-(z_3-11)/1 & \text{if } 11 \leq z_3 \leq 12 \\ 1 & \text{if } z_3 \leq 11 \end{cases}$$

A fuzzy multi-objective linear program with max-min approach is given as

max □
s.t.

$$\begin{aligned}
&\square \leq 1-(z_1-65)/5 \\
&\square \leq 1-(z_2-16)/2 \\
&\square \leq 1-(z_3-11)/1 \\
&X_{01}+X_{02}+X_{03} = 1 \\
&X_{10}+X_{12}+X_{13} = 1 \\
&X_{20}+X_{21}+X_{23} = 1 \\
&X_{30}+X_{31}+X_{32} = 1 \\
&X_{10}+X_{20}+X_{30} = 1 \\
&X_{01}+X_{21}+X_{31} = 1 \\
&X_{02}+X_{12}+X_{32} = 1 \\
&X_{03}+X_{13}+X_{23} = 1 \\
&X_{01}+X_{10} \leq 1 \\
&X_{02}+X_{20} \leq 1 \\
&X_{03}+X_{30} \leq 1 \\
&X_{12}+X_{21} \leq 1 \\
&X_{13}+X_{31} \leq 1 \\
&X_{23}+X_{32} \leq 1 \\
&\square \geq 0 \\
&X_{ij} \geq 0
\end{aligned}$$

The above fuzzy linear program and its variants were solved by using TORA. As shown in Table-2 when only $z_1$, and $z_2$ are considered, and $z_3$ is omitted, an optimal rout with $\alpha$ = 0.8 is yielded. When $z_3$ is also considered, the solution becomes infeasible on these tolerances. Again by relaxing tolerance in $z_3$ to 4, solution becomes feasible. In this case, the optimal path is achieved with $\alpha$ = 0.55. By increasing the tolerance in $z_3$ from 4 to 5, an optimal solution with $\alpha$ = 0.62 is obtained. These results show that by adjusting tolerance, we can determine optimal solution to a multi criteria TSP.

**Table 2:**

| Sol. | $Z_1,t_1$ | $Z_2,t_2$ | $Z_3,t_3$ | $\alpha$ | Route |
|---|---|---|---|---|---|
| **1** | 65,5 | 16,2 | Not | 0.80 | $x_{03}$-$x_{31}$-$x_{20}$ |
| **2** | 65,5 | 16,2 | 11,1 | ---- | no feasible sol |
| **2** | 65,5 | 16,2 | 11,4 | 0.55 | $x_{03}$-$x_{31}$-$x_{12}$-$x_{20}$ |
| **3** | 65,5 | 16,2 | 11,5 | 0.62 | $x_{03}$-$x_{31}$-$x_{12}$-$x_{20}$ |

## 5. Conclusion

The focus of this paper is the analysis of the symmetric TSP as a Fuzzy problem with vague decision parameters. What general lesson can be taken from this study is: First multi objective TSP exists in uncertain or vague environment where route selection is done by exploiting these parameters. Second, the tolerances are introduced by the decision maker to accommodate this vagueness. Third, by adjusting these tolerances, a range of solutions with different aspiration level are found from which decision maker can choose the one that best meets his satisfactory level within the given domain of tolerances.

Fuzzy multi-objective linear programming can be helpful in order to achieve the k-dimension points according to the decision maker aspiration level in multi-dimension solution space.

## 6. Future work

Fuzzy multi-objective linear programming and TSP are rich enough to cater a lot of future research in the field of operations research and artificial intelligence. There is definite potential for work on development of methods to solve TSP problems with vague description of resources. For efficient results, there is need of some heuristics e.g. relative dependencies among objective function can also be determined. We are in progress to develop a software tool for simulation of the proposed approach.

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