A catastrophic-cum-restorative queuing system with correlated batch arrivals and general service time distribution

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Abstract

In this paper, a stochastic queuing model for a catastrophic-cum-restorative queuing system with correlated batch arrivals and general service time distribution has been developed. The transient analysis of the queuing model has been performed. The Laplace Transform of the probability generating function of the system size has been obtained. Finally, some particular cases of the model have been derived and discussed.

Keywords: Queue length, Catastrophes, Broadband services, General service time distribution, and Restoration.

1. Introduction

Broadband Communication Networks are playing a key role in providing a variety of multimedia services such as voice, video and data etc. The amount of information per unit time generated by these services varies along the connection duration. There are certain periods in which the information rate increases and others in which it decreases or becomes null. As the sources providing such services are not synchronized several cells may arrive at the same slot. Thus, they (cells) arrive in batches of variable size. Parra (1993) studied that the arrival process in broadband communication networks is correlated in nature.

Further, the arrival of infected cells (viruses) and noise bursts etc. may annihilate all the cells in the buffer of the server (computer) and leave it momentarily inactivated until the new cell arrival occurs. Such infected cells may be modeled by catastrophes. The notion of catastrophes occurring at random, leading to annihilation of all the customers there and the momentary inactivation of service facility until a new arrival of a customer is not uncommon in many practical problems. Chao (1995) studied a queuing network model with catastrophes. Crescenzo et al. (2003) studied an M/M/1 queue with catastrophes and derived its heavy traffic approximation. Jain and Kumar (2004, 2005, 2006) obtained the transient solution of some catastrophic queuing systems with correlated input. Murari (1972) obtained the time dependent solution of a queuing problem with correlated batch arrivals and general service time distribution. Kumar (2008) obtained the time-dependent solution of a catastrophic-cum-restorative queuing problem having correlated batch input and variable service capacity. Kumar (2009) studied a correlated input queue with catastrophic and restorative effects for the cell traffic generated by new broadband services. Recently, Kumar (2010) studied an M/M/2 queue with heterogeneous servers under catastrophic and restorative effects and obtained its transient solution.

The concept of catastrophe has tremendous applications in a wide variety of areas particularly in computer-communication, biosciences, population studies and industries etc. It is based on the assumption that with the occurrence of catastrophe, all the customers in the system are destroyed and simultaneously the system becomes ready to accept new customers. However a system will always require some sort of time to function in a normal way if it suffers from catastrophe, which is taken as restoration time. Thus, it would be more practicable if we model the restoration time required by a system which is suffering from catastrophe. In the present example, with the occurrence of catastrophe all the cells in the buffer of the server are destroyed immediately. But the server can work properly after it is free from the viruses and noise bursts. Thus, some sort of recovery / restoration time is needed. To this end, the concept of restoration time has been introduced in which no arrival is allowed to occur.

In this paper, we incorporate the effect of catastrophes and restoration in the correlated batch arrival queue with general service time distribution. We consider a single server catastrophic-cum-restorative queuing system with correlated batch input and general service time distribution. The transient solution of the model under investigation has been derived.

This paper has been organized as follows: In section 2, the queuing model has been formulated. In section 3, the transient solution of the model has been obtained.

2. Formulation of Queuing Model

The queuing model investigated in this paper is based on the following assumptions:

1. The customers arrive at a service facility in batches, the size of the batch being a random variable with

   Prob. (size of the batch is j) = c_j , j=1,2,3,…

   and

   $$\sum_{j=1}^{\infty} c_j = 1$$

2. The arrival of a batch can occur only at the transition marks t_0, t_1, t_2… where $\theta_r = t_r - t_{r-1}$; r = 1,2,3,…are random variables with $P[\theta_r \le x] = 1 - \exp(-\lambda x)$; $\lambda > 0$, r = 1,2,…

   The arrival and no arrival of a batch at two consecutive transition marks t_{r-1}, t_r; r=1,2,3,...are governed by the following transition probability matrix:

From t_{r-1} \ To t_r	arrival of a batch	no arrival of a batch	
arrival of a batch	p_{11}	p_{10}	where $p_{11}+p_{10}=1$ and
no arrival of a batch	p_{01}	p_{00}	$p_{01}+p_{00}=1$

Thus, the arrivals of batches at two consecutive transition marks are correlated.

3. The queue discipline is first-come-first-served.
4. The service time distribution is general with probability distribution function D(x).
5. When the system is not empty, the catastrophes occur at the service facility according to a Poisson process of rate ξ. The catastrophes annihilate all the customers in the system instantaneously.
6. The restoration times are independently, identically and exponentially distributed with parameter η.
7. The stochastic processes namely (i) distributions of θ_r, r=1,2,3,…(ii) the Distribution of the size of the batch (iii) service time distribution (iv) the distribution of catastrophes, and (v) the distribution of restoration times are independent of each other.
8. Let the time be reckoned from the instant when the service channel is idle and a transition with no arrival of a batch has just occurred, so that $Q_{0,0}(0)=1$

Define,

$P_{n,0}(x,t)dx$ = the probability that at time t, the queue length (the number of customers waiting excluding those being served) is equal to n, a unit being served with elapsed service time lying between x and x + dx, and no arrival of a batch has occurred at the previous transition mark.

$P_{n,1}(x,t)dx$ = the probability that at time t, the queue length is equal to n, a unit being served with elapsed service time lying between x and x + dx, and an arrival of a batch has occurred at the previous transition mark.

$P_{n,0}(t)$ = the probability that at time t, the queue length is equal to n, the service channel is not idle and no arrival of a batch has occurred at the previous transition mark.

$P_{n,1}(t)$ = the probability that at time t, the queue length is equal to n, the service channel is not idle and an arrival of a batch has occurred at the previous transition mark.

$Q_{0,0}(t)$ = the probability that at time t, the queue length is equal to 0 without the occurrence of catastrophe, the service channel is idle and no arrival of batch has occurred at the previous transition mark.

$C_{0,0}(t)$ = the probability that at time t, the queue length is equal to 0 with the occurrence of catastrophe, the service channel is idle and no arrival of batch has occurred at the previous transition mark.

$Q_{0,1}(t)$ = the probability that at time t, the queue length is equal to 0 without the occurrence of catastrophe, the service channel is idle and an arrival of a batch has occurred at the previous transition mark.

$C_{0,1}(t)$ = the probability that at time t, the queue length is equal to 0 with the occurrence of catastrophe, the service channel is idle and an arrival of a batch has occurred at the previous transition mark.

$R_n(t)$ = the probability that at time t, the queue length is equal to n.

3. Transient Solution of the Model

The governing equations of the model are:-

$$R_n(t)=P_{n,0}(t)+P_{n,1}(t) \quad ;n=1,2,3\ldots \tag{1}$$

$$R_0(t) = P_{0,0}(t) + P_{0,1}(t) + Q_{0,0}(t) + Q_{0,1}(t) \tag{2}$$

$$P_{n,i}(t)=\int_0^\infty P_{n,i}(x,t)dx \; ; \quad i=0,1 \tag{3}$$

$$\frac{d}{dt}Q_{0,0}(t) = -\lambda Q_{0,0}(t) + \int_0^\infty P_{0,0}(x,t)\beta(x)dx + \lambda[p_{00} Q_{0,0}(t) + p_{10} Q_{0,1}(t)] + \eta C_{0,0}(t) \tag{4}$$

$$\frac{d}{dt}C_{0,0}(t) = -\eta C_{0,0}(t) + \xi\left[\sum_{n=0}^{n=\infty} P_{n,o}(x,t)\right] \tag{5}$$

$$\frac{d}{dt}Q_{0,1}(t)=-\lambda Q_{0,1}(t)+\int_0^\infty P_{0,1}(x,t)\beta(x)dx+\eta C_{0,1}(t) \tag{6}$$

$$\frac{d}{dt}C_{0,1}(t) = -\eta C_{0,1}(t) + \xi\left[\sum_{n=0}^{n=\infty} P_{n,1}(x,t)\right] \tag{7}$$

$$\frac{\partial}{\partial x}P_{n,0}(x,t)+\frac{\partial}{\partial t}P_{n,0}(x,t) = -(\lambda+\beta(x)+\xi)P_{n,0}(x,t) + \lambda[p_{00} P_{n,0}(x,t)+p_{10} P_{n,1}(x,t)] \quad ; n=1,2,3, \tag{8}$$

$$\frac{\partial}{\partial x}P_{n,1}(x,t)+\frac{\partial}{\partial t}P_{n,1}(x,t) = -(\lambda+\beta(x)+\xi)P_{n,1}(x,t) + \lambda\sum_{j=1}^{n} c_j [p_{01} P_{n-j,0}(x,t)+p_{11} P_{n-j,1}(x,t)] \quad ;n=1,2,3, \tag{9}$$

$$\frac{\partial}{\partial x}P_{0,1}(x,t)+\frac{\partial}{\partial t}P_{0,1}(x,t) = -(\lambda+\beta(x)+\xi)P_{0,1}(x,t) \quad (10)$$

where β(x)dx is the first order probability that the service will be completed in time interval (x, x+dx) conditioned that the same had not been completed till time x, and is related to D(x) by the relation

$$D(x) = \beta(x)e^{-\int_0^x \beta(x)dx} \quad (11)$$

These equations are to be solved subject to the following boundary conditions:

$$P_{n,0}(0,t)=\int_0^\infty P_{n+1,0}(x,t)\beta(x)dx \quad (12)$$

$$P_{n,1}(0,t)=\int_0^\infty P_{n+1,1}(x,t)\beta(x)dx+\lambda c_{n+1}[p_{01}Q_{0,0}(t)+p_{11}Q_{0,1}(t)] \quad (13)$$

Define, the Laplace Transform (L. T.) of f (t) by

$$f^*(s)=\int_0^\infty e^{-st}f(t)dt$$

Taking L.T.'s of (1) - (10), we have

$$R^*_n(s) = P^*_{n,0}(s) + P^*_{n,1}(s); \qquad n=1,2,3, \quad (14)$$

$$R^*_0(s) = P^*_{0,0}(s) + P^*_{0,1}(s) + Q^*_{0,0}(s) + Q^*_{0,1}(s) \quad (15)$$

$$P^*_{n,i}(s)=\int_0^\infty P^*_{n,i}(x,s)dx; \qquad i=0,1 \quad (16)$$

$$(s+\lambda)Q^*_{0,0}(s)-1=\int_0^\infty P^*_{0,0}(x,s)\beta(x)dx+\lambda[p_{00}Q^*_{0,0}(s)+p_{10}Q^*_{0,1}(s)]+\eta C^*_{0,0}(s) \quad (17)$$

$$(s+\eta)C^*_{0,0}(s) = \xi\left[\sum_{n=0}^{n=\infty}P^*_{n,o}(x,s)\right] \quad (18)$$

$$(s+\lambda)Q^*_{0,1}(s) = \int_0^\infty P^*_{0,1}(x,s)\beta(x)dx+\eta C^*_{0,1}(s) \quad (19)$$

$$(s+\eta)C^*_{0,1}(s) = \xi\left[\sum_{n=0}^{n=\infty}P^*_{n,1}(x,s)\right] \quad (20)$$

$$\frac{\partial}{\partial x}P^*_{n,0}(x,s)+(s+\lambda+\beta(x)+\xi)P^*_{n,0}(x,s) = \lambda[p_{00}P^*_{n,0}(x,s)+p_{10}P^*_{n,1}(x,s)]$$

; n =1,2,3, (21)

$$\frac{\partial}{\partial x}P^*_{n,1}(x,s)+(s+\lambda+\beta(x)+\xi)P^*_{n,1}(x,s)= \lambda\sum_{j=1}^{n}c_j\,[\,p_{01}\,P^*_{n-j,0}(x,s)+p_{11}\,P^*_{n-j,1}(x,s)]$$

;n =1,2,3,… (22)

$$\frac{\partial}{\partial x}P^*_{0,1}(x,s)+(s+\lambda+\beta(x)+\xi)P^*_{0,1}(x,s)=0 \qquad (23)$$

Define, the following probability generating functions by

$$P^*_i(x,s,\alpha) = \sum_{n=0}^{n=\infty}\alpha^n P^*_{n,i}(x,s), i=0,1 \qquad (24)$$

$$P^*_i(s,\alpha) = \sum_{n=0}^{n=\infty}\alpha^n P^*_{n,i}(s) \text{ ,i=0,1} \qquad (25)$$

$$R^*(s,\alpha) = \sum_{n=0}^{n=\infty}\alpha^n R^*_n(s) \qquad (26)$$

Multiplying (14) and (15) by appropriate powers of α and adding, we have

$$R^*(s,\alpha) = P^*_0(s,\alpha) + P^*_1(s,\alpha) + Q^*_{0,0}(s) + Q^*_{0,1}(s) \qquad (27)$$

Similarly, from (16) we have

$$P^*_i(s,\alpha) = \int_0^\infty P_i^*(x,s,\alpha)dx \qquad (28)$$

(21) gives

$$\frac{\partial}{\partial x}P^*_0(x,s,\alpha)+[s+\lambda+\beta(x)+\xi-\lambda p_{00}]P^*_0(x,s,\alpha)=\lambda p_{10}P^*_1(x,s,\alpha) \qquad (29)$$

and (22) and (23) yield

$$\frac{\partial}{\partial x}P^*_1(x,s,\alpha)+[s+\lambda+\beta(x)+\xi-\lambda C(\alpha)p_{11}]P^*_1(x,s,\alpha)=\lambda C(\alpha)p_{01}P^*_0(x,s,\alpha) \qquad (30)$$

Where $C(\alpha)=\sum_{n=1}^{\infty}\alpha^n c_n$

Substituting for $P^*_1(x, s,\alpha)$ from (29) in (30), we get

$$\frac{\partial^2}{\partial x^2}P^*_0(x,s,\alpha)+\left[2(s+\lambda+\beta(x)+\xi)-\lambda(p_{00}+C(\alpha)p_{11})\right]\frac{\partial}{\partial x}P^*_0(x,s,\alpha)+$$
$$\left[\frac{d}{dx}\beta(x)-\lambda^2C(\alpha)p_{10}p_{01}+[s+\lambda+\beta(x)+\xi-\lambda C(\alpha)p_{11}][s+\lambda+\beta(x)+\xi-\lambda p_{00}]\right]P^*_0(x,s,\alpha)=0$$

(31)

Solving this differential equation we have

$$P^{*}{}_{0}(x,s,\alpha)=\left[A_{1}e^{\lambda ax}+B_{1}e^{-\lambda ax}\right]U(x) \tag{32}$$

$$U(x)=\exp\left[\frac{\lambda}{2}\left[p_{00}+C(\alpha)p_{11}\right]x-(s+\lambda+\xi)x-\int_{0}^{x}\beta(x)dx\right] \tag{33}$$

$$a^{2}=C(\alpha)p_{10}p_{01}+\left[\frac{p_{00}-C(\alpha)p_{11}}{2}\right]^{2} \tag{34}$$

Where A_1, B_1 are constants of integration.

Substituting for $P^{*}{}_{0}(x, s,\alpha)$ from (32) in (29), we have

$$P^{*}{}_{1}(x,s,\alpha)=\left[(a-b)A_{1}e^{\lambda ax}-(a+b)B_{1}e^{-\lambda ax}\right]\frac{U(x)}{p_{10}} \tag{35}$$

Where

$$b=\frac{p_{00}-C(\alpha)p_{11}}{2} \tag{36}$$

Setting x= 0 in (32) and (35) and then solving simultaneously, we have

$$A_{1}=\left[\frac{(a+b)P^{*}{}_{0}(0,s,\alpha)+p_{10}P^{*}{}_{1}(0,s,\alpha)}{2a}\right] \tag{37}$$

$$B_{1}=\left[\frac{(a-b)P^{*}{}_{0}(0,s,\alpha)-p_{10}P^{*}{}_{1}(0,s,\alpha)}{2a}\right] \tag{38}$$

Therefore, we have

$$P^{*}{}_{0}(x,s,\alpha)=\frac{1}{2a}\left[(a+b)P^{*}{}_{0}(0,s,\alpha)+p_{10}P^{*}{}_{1}(0,s,\alpha)\right]\exp\left[-h_{1}x-\int_{0}^{x}\beta(x)dx\right]+\frac{1}{2a}\left[(a-b)P^{*}{}_{0}(0,s,\alpha)-p_{10}P^{*}{}_{1}(0,s,\alpha)\right]\exp\left[-h_{2}x-\int_{0}^{x}\beta(x)dx\right] \tag{39}$$

$$P^{*}{}_{1}(x,s,\alpha)=\frac{1}{2a}\left[(a-b)P^{*}{}_{1}(0,s,\alpha)+C(\alpha)p_{01}P^{*}{}_{0}(0,s,\alpha)\right]\exp\left[-h_{1}x-\int_{0}^{x}\beta(x)dx\right]+\frac{1}{2a}\left[(a+b)P^{*}{}_{1}(0,s,\alpha)-C(\alpha)p_{01}P^{*}{}_{0}(0,s,\alpha)\right]\exp\left[-h_{2}x-\int_{0}^{x}\beta(x)dx\right] \tag{40}$$

Where

$$h_{1}=s+\xi+\lambda(p_{01}+b-a)$$

$$h_{2}=s+\xi+\lambda(p_{01}+b+a)$$

Substituting for $P^*{}_0(x,s,\alpha)$ and $P^*{}_1(x,s,\alpha)$ from (39) and (40) in (28) and using (27), we get

$$\begin{aligned} R^*(s,\alpha) &= Q^*{}_{0,0}(s) + Q^*{}_{0,1}(s) + \\ &+ \frac{1}{2a}\left[(a+b+C(\alpha)p_{01})P^*{}_0(s,\alpha) + (a-b+p_{10})P^*{}_1(0,s,\alpha)\right]\frac{1-D^*(h_1)}{h_1} + \\ &+ \frac{1}{2a}\left[(a-b-C(\alpha)p_{01})P^*{}_0(s,\alpha) + (a+b-p_{10})P^*{}_1(0,s,\alpha)\right]\frac{1-D^*(h_2)}{h_2} \end{aligned} \quad (41)$$

Multiplying (12) by α^n, summing over 0 to ∞ and taking Laplace Transform, we have

$$\alpha P^*{}_0(0,s,\alpha) = \int_0^\infty P^*{}_0(x,s,\alpha)\beta(x)dx + K_0 \quad (42)$$

Where

$$K_0 = -\int_0^\infty P^*{}_{0,0}(x,s)\beta(x)dx \quad (43)$$

Similarly, (13) gives

$$\alpha P^*{}_1(0,s,\alpha) = \int_0^\infty P^*{}_1(x,s,\alpha)\beta(x)dx + K_1 \quad (44)$$

Where

$$K_1 = \lambda C(\alpha)\left[Q^*{}_{0,0}(s)p_{01} + Q^*{}_{0,1}(s)p_{11}\right] - \int_0^\infty P^*{}_{0,1}(x,s)\beta(x)dx \quad (45)$$

Substituting for $P^*{}_i(x,s,\alpha)$ from (39) and (40) in (42) and (44), we have

$$\begin{aligned} K_0 &= \left[\alpha - D^*(h_2)\right]P^*{}_0(0,s,\alpha) - \frac{1}{2a}\left[D^*(h_1) - D^*(h_2)\right] \\ &\quad \left[(a+b)P^*{}_0(0,s,\alpha) + p_{10}P^*{}_1(0,s,\alpha)\right] \end{aligned} \quad (46)$$

$$\begin{aligned} K_1 &= \left[\alpha - D^*(h_2)\right]P^*{}_1(0,s,\alpha) - \frac{1}{2a}\left[D^*(h_1) - D^*(h_2)\right] \\ &\quad \left[(a-b)P^*{}_1(0,s,\alpha) + C(\alpha)p_{01}P^*{}_0(0,s,\alpha)\right] \end{aligned} \quad (47)$$

Solving (46) and (47) for $P^*{}_i(0,s,\alpha)$, we have

$$P^*{}_i(0,s,\alpha) = \frac{N_i}{E}, i = 0,1 \quad (48)$$

Where

$$N_0 = \left[\alpha - D^*(h_2)\right]K_0 - \frac{1}{2a}\left[D^*(h_1) - D^*(h_2)\right] \left[(a-b)K_0 - p_{10}K_1\right] \tag{49}$$

$$N_1 = \left[\alpha - D^*(h_2)\right]K_1 - \frac{1}{2a}\left[D^*(h_1) - D^*(h_2)\right] \left[(a+b)K_1 - C(\alpha)p_{01}K_0\right] \tag{50}$$

$$E = \left[\alpha - D^*(h_1)\right]\left[\alpha - D^*(h_2)\right] \tag{51}$$

Combining (41) and (48), we get

$$R^*(s,\alpha) = \left[Q^*{}_{0,0}(s) + Q^*{}_{0,1}(s)\right] + \frac{1 - D^*(h_1)}{2ah_1}\frac{\left[a+b+C(\alpha)p_{01}\right]K_0 + \left[a-b+p_{01}\right]K_1}{\alpha - D^*(h_1)} + \frac{1 - D^*(h_2)}{2ah_2}\frac{\left[a-b-C(\alpha)p_{01}\right]K_0 + \left[a+b-p_{10}\right]K_1}{\alpha - D^*(h_2)} \tag{52}$$

Also, $P^*{}_0(x,s,\alpha)$ and $P^*{}_1(x,s,\alpha)$ from (39) and (40) for α=1 give the values of the summations $\sum_{n=0}^{n=\infty} P^*{}_{n,o}(x,s)$ and $\sum_{n=0}^{n=\infty} P^*{}_{n,1}(x,s)$ respectively.

Also from (18) and (20), we have

$$C^*{}_{0,0}(s) = \left(\frac{\xi}{s+\eta}\right)\left[\sum_{n=0}^{n=\infty} P^*{}_{n,o}(x,s)\right] \tag{53}$$

and

$$C^*{}_{0,1}(s) = \left(\frac{\xi}{s+\eta}\right)\left[\sum_{n=0}^{\infty} P^*{}_{n,1}(x,s)\right] \tag{54}$$

Thus, substituting the values of $C^*{}_{0,0}(s)$ and $C^*{}_{0,1}(s)$ in (17) and (19) we get two equations in four unknowns.

By Rouche's theorem $\alpha - D^*(h_1)$ has exactly one root inside the unit circle $|\alpha| < 1$. Since $R^*(s,\alpha)$ is analytic inside the unit circle, this root must vanish the numerator of the second term of the right hand side of (52), giving rise to one equation. Similarly, one root of $\alpha - D^*(h_2)$ must vanish the numerator of the third term of the right hand side of (52) giving rise to one equation. Solving, these two equations along with (17) and (19), one can determine all the four unknowns viz.

$Q^*_{0,0}(s), Q^*_{0,1}(s)$, $\int_0^\infty P^*_{0,0}(x,s)\beta(x)dx$ and $\int_0^\infty P^*_{0,1}(x,s)\beta(x)dx$ occurring in K_0 and K_1.

Thus, $R^*(s,\alpha)$ is completely determined.

Particular Cases

(1) When the service time is exponential with parameter μ, then

$D^*(h_1) = \frac{\mu}{\mu + h_1}$ and $D^*(h_2) = \frac{\mu}{\mu + h_2}$

and from (52), we have

$$R^*(s,\alpha) = Q^*_{0,0}(s) + Q^*_{0,1}(s) + \frac{\left[h + \lambda\alpha^2(p_{01} - p_{11})K_0\right] + \left[h + \lambda\alpha(p_{10} - p_{00})K_1\right]}{(h - \lambda\alpha p_{00})(h - \lambda\alpha^2 p_{11}) - \lambda^2\alpha^3 p_{01} p_{10}} \quad (55)$$

Where $h = (s + \lambda + \mu + \xi)\alpha - \mu$ and

$K_0 = -\mu P^*_{0,0}(s)$

$K_1 = \alpha + \lambda\alpha[p_{01}Q^*_{0,0}(s) + p_{11}Q^*_{0,1}(s)] - \mu P^*_{0,1}(s)$

That is, the model reduces to a catastrophic-cum-restorative queuing system with correlated batch input and exponential service time distribution.

(2) When $\xi=0$ and $\eta = \infty$ (i.e. there are no catastrophic and restorative effects), from (52)-(54), we have

$$R^*(s,\alpha) = \left[Q^*_{0,0}(s) + Q^*_{0,1}(s)\right] + \frac{1 - D^*(h_1)}{2ah_1} \frac{\left[a + b + C(\alpha)p_{01}\right]K_0 + \left[a - b + p_{01}\right]K_1}{\alpha - D^*(h_1)} + \frac{1 - D^*(h_2)}{2ah_2} \frac{\left[a - b - C(\alpha)p_{01}\right]K_0 + \left[a + b - p_{10}\right]K_1}{\alpha - D^*(h_2)} \quad (56)$$

Where

$h_1 = s + \lambda(p_{01} + b - a)$

$h_2 = s + \lambda(p_{01} + b + a)$

$$C^*_{0,0}(s) = C^*_{0,1}(s) = 0 \quad (57)$$

(56) provides us the Laplace transform of the probability generating function of system size of a correlated batch input queue having general service time distribution studied by Murari (1972).

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Conclusion

A correlated batch input, single server queuing system with catastrophic and restorative effects has been studied. The service time distribution has been taken as general. The transient analysis of the queuing model has been carried out. The Laplace transform of the probability generating function of system size has been obtained. Two particular cases of the model have been discussed. The limitation of the queuing model studied in this paper is that the explicit expressions for time-dependent probabilities have not been obtained.

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